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THE CONTRIBUTION OF A RADICAL ANTI-NEOCLASSICAL. OSKAR MORGENSTERN AND THE HETERODOX POTENTIALITIES OF THE APPLICATION OF GAME THEORY TO ECONOMICS

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ABSTRACT. Up until the early sixties the application of game theory to economics was confined to the introduction into general equilibrium models of some new logical and mathematical tools. The consequent emphasis on formal aspects has delayed an interpretation of the theory of games more in accordance with the purposes explaining its creation and, particularly, with the necessity of removing the principal simplifications of the Walrasian system. These heterodox potentialities were already implicit in Oskar Morgenstern's criticism of neoclassical economics and seem to emerge again from some recent research areas that, by applying game theory, seek to give up the principal neoclassical postulate, the identity between rational choice and the solution of a well defined maximum problem.

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Introduction

In its brief history, the relation between game theory and economics has been characterized by phases of feverish elaboration of new contributions followed by periods in which an open scepticism concerning its usefulness prevailed. Moreover, this vaguely cyclical pattern of evolution has affected various research areas; after an initial period in which game theorists had focused their attention on competitive market models, in the 1970s and 1980s they especially turned to the problems of oligopolistic markets and bargaining; today the latter research areas are going through a phase of sedimentation and selection while other fields of study receive a strong impulse by the application of game theory.

In order to understand the causes of this particular development, it is useful to analyze the period of introduction of game theory into economics, following the publication of von Neumann and Morgenstern's *Theory of Games and Economic Behavior* (1944). This fundamental book gave two important contributions, distinctly imputable to its authors: the first, attributable to von Neumann and regarding the logical-mathematical instrumentation of economists, has been widely discussed and appreciated; the other, attributable to Oskar Morgenstern, has attracted minor attention, although it is directly concerned with economic theory.

Such emphasis on formal aspects of game theory characterized its application to economics at least until the '70s, when the identification with competitive markets was attenuated. But, in the meanwhile, this fact has had an important consequence: the delay of an interpretation of game theory more in accordance with the purposes explaining its creation and, particularly, with the necessity of removing the principal simplifications of the Walrasian system.

Developing these premises, this paper intends to demonstrate that these heterodox potentialities of game theory were already implicit in Oskar Morgenstern's severe criticism of neoclassical theory, contained in some works published in the '30s and '40s. Founding the relation between game theory and economics on these ideas, rather than on those characterizing the application of von Neumann and Nash's new mathematical tools to general equilibrium models, would have avoided its identification with a concept of *strong* rationality. Moreover, it argues that today this interpretation of game theory emerges again in some recent fields of study, that try to give up the most

important neoclassical postulate, the identity between rational choice and the solution of a well defined maximum problem..

The paper is essentially divided into three parts. Part one deals with the first economic applications of game theory and shows that in the 50's general equilibrium models gave a limited and misleading interpretation of the new method. The second part presents Oskar Morgenstern's criticism of neoclassical theory, contained in some works published between 1928 and 1948, and the proposals included in the same literature. The last part gives a brief description of some recent economic theories that try to weaken or to abandon the neoclassical postulate of maximization, reviving a fundamental component of Morgenstern's original project.

The early years of the application of game theory to economics: the orthodox reading

Historical studies on the introduction of game theory into economics agree on at least two general remarks: the first is that the predecessors of *Theory of Games and Economic Behavior* are not directly imputable to the history of economic thought and, therefore, von Neumann and Morgenstern were the first authors to propose a systematic application of game theory to economics; the second is that we can give them credit for having created a new language to represent concepts and principles already known in economics.

If these considerations confirm that the book's objective, as emerged from its first pages[1], was reached, they do not however point out that *Theory of Games* represented a turning point for two different aspects, distinctly imputable to its two authors. Morgenstern was the radical critic of neoclassical economics and the upholder of its overcoming through game theory; von Neumann's ingenious mathematical ideas deserved the merit of having modified the economists' tool box, introducing axiomatic method and modern mathematics in it.

In the '50s these two research programmes, distinguished only for historical necessity but taken as a whole in *Theory of Games*, had a different impact on economic analysis. Few authors immediately comprehended the unity characterizing von Neumann and Morgenstern's book; Shubik's oligopolistic market analysis and

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1 "The purpose of this book is to present a discussion of some fundamental questions of economic theory which require a treatment different from that which they have found thus far in the literature. The analysis is concerned with some basic problems arising from a study of economic behavior which have been the centre of attention of economists for a long time." (von Neumann and Morgenstern 1944, 1).

Harsanyi's bargaining theory[2] were two examples of contributions using game theory to overcome the restrictive hypotheses of the neoclassical school, even if a full development of their results was not to be achieved until the 1970's. But in competitive markets analysis founded on general equilibrium models, von Neumann's program was the only to be applied. The principal outcome of this literature, Arrow and Debreu's proof of the existence of an equilibrium for a competitive economy, was a meaningful example of this halved relation between economics and game theory .

In 1954, Arrow and Debreu published one of the most important paper of contemporary economics. Its contents represent the final act of a long story started with Walras' general equilibrium model and carried on by various attempts to mathematically improve the original proof of existence[3]. So as to give a satisfactory treatment of such a central problem for neoclassical school, Arrow and Debreu employed Kakutani's fixed point theorem (1941), used by Nash (1950) to prove the existence of an equilibrium in *n*-person non-cooperative games, and the theory of games. Their proof is founded on the concept of *abstract economy*, corresponding to a generalization of a game:

> An abstract economy, then, may be characterized as a generalization of a game in which the choice of an action by one agent affects both the pay-off and the domain of actions of other agents. (Arrow and Debreu 1954, 273)

This original concept is still only applied to the construction of the demand function; while in the previous models the consumer's decision was the result of the utility maximization, given prices and income, in Arrow and Debreu's paper these latter values are expressed in function of the consumers' choices. On the contrary, the description of the productive units' behavior lacks any reference to strategic interaction and the proof of the existence of general equilibrium is founded on the artificial Walrasian *tâtonnement*. The *abstract economy* corresponds indeed to a $m+n+1$-person game, in which m consumers choose a strategy from a finite set of alternative consume vectors receiving a payoff in terms of utility, n productive units adopt a production vector obtaining a profit and the *market participant* - the Walrasian auctioneer - determines price equilibrium vector.

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[2] Shubik's work is collected in *Strategy and Market Structure* (1959), developing his own Ph.D. thesis, *Competition and the Theory of Games* (1953). Harsanyi's work is contained in a 1956 *Econometrica* article, giving the proof of the mathematical equivalence between Nash's and Zeuthen's solution to the wage bargaining. For an history of the introduction of game theory into economics from the origins to 1959, see my dissertation (Innocenti 1993).

[3] This history is the object of a wide and good literature, among which one can mention Weintraub's *The Journal of Economic Literature* paper, Ingrao and Israel's book on the *invisible hand*, the two volumes edited by Feiwel and dedicated to Kenneth J. Arrow and Punzo's recent article on the Viennese Circle (Weintraub 1983, Ingrao and Israel 1987, Feiwel 1987a e 1987b, Punzo 1991).

The only other meaningful relation between Arrow and Debreu's proof and game theory consists in the application of Nash equilibrium, that corresponds to competitive equilibrium in the $m+n+1$-person game and allows proving the following theorems:

a) if each player possesses an initial endowment of each marketable commodity, then the game has an equilibrium point;

b) in presence of labor, an equilibrium point exists if the model includes some types of labor with the properties that every player must supply a positive quantity of at least one type of work and every type of work must have a positive utility in the production of the commodities.

According to this view, it is Kakutani's fixed point theorem, in the version applied by Nash, that permits the solution of the model, as Arrow himself acknowledges in a book edited by George Feiwel:

> I read, first von Neumann, but especially Nash's 1950 paper. It suddenly struck me: «This is very much like the problem of competitive equilibrium». I thought about it on and off, until one day, when I had a few free hours, I thought how to interpret competitive equilibrium as a game. After a number of steps, you can take Nash's result and apply it. (Arrow 1987, 194)

But also the concept of game is used only because it allows applying a new mathematical tool rather than representing strategic interaction situations[4].

Even the following developments confirmed the marginal relevance of game theory in competitive models. In 1959 Debreu published the *Theory of Value*, which can be considered the most systematic and complete expression of neoclassical theory. In this work, Debreu renounces the concept of *abstract economy* and returns to the original Walrasian formulation[5]. The only credit he attributes to von Neumann e Morgenstern is that of having "freed mathematical economics from its traditions of differential calculus and compromises with logic" (Debreu 1959, VIII) and of creating a new mathematical economics founded on topology and convex analysis[6], but evidently not that of having threatened the validity of neoclassical theory. The first six chapters of

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[4] This view is indirectly confirmed by Lionel McKenzie, who obtained the same proof of existence without using game theory: "Gerard Debreu and Kenneth Arrow have been working, independently, along similar lines. Their method seems closely related to the theory of competitive games developed by John Nash, while my motivation comes directly from the work of Abraham Wald and Tjialling Koopmans." (McKenzie 1954, 147). However, McKenzie's proof applies Kakutani's fixed point theorem as well.

[5] See Ingrao and Israel 1987, 286.

[6] See Tani 1987, 79.

the *Theory of Value* lack any reference to game theory and the proof of the existence of competitive equilibrium postulates an exogenous price system again. The only reference appears in chapter 7, where the choice of an economic agent in condition of uncertainty is represented through a game in extensive form.

In the 1960s, the analysis of the uniqueness and stability of competitive equilibrium avoided almost any reference to the new tool[7]. The identification between the application of game theory to economics and the analysis of competitive markets was fairly sustained by the discovery of a strict relation between a concept for the solution of cooperative games, the *core*[8], and Edgeworth's contract curve (1881). It was Shubik (1959b) who recovered the Edgeworthian model of a symmetric market and to show, firstly, that the *core* is equivalent to the contract curve although it does not necessarily tend to a single point in presence of coalitions among players and secondly to define a set of conditions assuring the existence and the uniqueness of the *core*. Later, this equivalence was analyzed by the better known Debreu and Scarf's paper (1963), proving it under more general conditions, and by Aumann's 1964 paper, representing a competitive economy through the concept of a *continuum* of agents and supplying a different proof of convergence between a game theoretical concept and a typical economic notion as the Walrasian equilibrium.

Even such contributions share the limits of Arrow and Debreu's approach, affirming a *formal* view of the relation between game theory and economics linked to the absence of real novelties from a theoretical point of view. In 1980, Schotter e Schwödiauer, tempting an assessment of the historical meaning of this literature, ascribed to it the function of having renewed the attention toward the economic applications of game theory. But, at the same time, they pointed out that the equivalence between those applications of game theory and Edgeworth's results had revived a standpoint already expressed by the reviewers of *Theory of Games*:

> While this result was quite elegant, it spelled the end of the first renaissance in game theory. It seemed that the game theoretical analysis (which employed strictly cooperative game theoretical concepts) was too demanding informationally to be of any intuitive appeal. Since it yielded no new results, little could be gained through its use. (Schotter and Schwödiauer 1980, 480)

On such a view, the impact of the first economic applications of game theory immediately exhausted itself and the ensuing crisis continued up until the early '70s.

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[7] For a discussion of this literature, see Ingrao e Israel (1987), chapters 11-12.

[8] The original formulation of the *core* is contained in two nearly contemporaneous works: D.B. Gillies, *Some Theorems on N-Person Games*, Department of Mathematics, Princeton University (Ph.D. Thesis), 1953 and L.S. Shapley, "Open Questions", in *Report of an Informal Conference on the Theory of N-Person Games*, Princeton University, mimeo, 1953.

Independently from the judgement about the more recent developments[9], Schotter and Schwödiauer's analysis corroborates two conclusions concerning the previous period:

a) in the initial phase, the few economic applications of game theory were principally concerned with general equilibrium models, while a full introduction of the new method in other research areas was postponed until the '70s[10];

b) due to their theoretical nature, these models applied only the logical-mathematical tools introduced by von Neumann, disregarding Oskar Morgenstern's contribution.

To do justice to this last inheritance, it is necessary, in the following section, to present a wide literature spanning almost twenty years from 1929 to 1948, which reaches its highest expression in the first chapter of the *Theory of Games*.

Oskar Morgenstern's criticism of neoclassical theory

A glance at the scientific careers of the authors of the *Theory of Games* can confirm that the economic ideas contained in that book are imputable to Oskar Morgenstern[11]. Before 1944, John von Neumann was involved in economics only thanks to his fundamental 1937 paper on growth which, even though not only formally innovative, assumed the principal neoclassical hypotheses[12]. On the contrary, Morgenstern became a social scientist who had written some papers, published first in Austria and then in United States, where he was forced to emigrate, containing a radical criticism of neoclassical theory. And it was just this contribution that provided a basis for the application of von Neumann's mathematical insights to economics.

So as to describe Morgenstern's criticism, it is useful to discuss each of its principal themes in turn ignoring the chronological sequence of their exposition. They

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[9] According to Schotter and Schwodiauer, the second renaissance is also imputable to Shubik's proof that general equilibrium can be represented through a set of isolated price-making agents (see Schotter and Schwödiauer 1980, 480).

[10] For a recent history of game theory, see the voice of *The New Palgrave* "Game Theory" (Aumann 1987).

[11] The 1992 Annual Supplement to *History of Political Economy*, edited by Weintraub and dedicated to the history of game theory, deals extensively with Morgenstern's contribution to *Theory of Games*. Schotter (1992, 96-103) and Leonard (1992, 51-57) share this view, while for Relistab (1992, 88-89) and Mirowski (1992, 143-144) Morgenstern's original contribution was not so important. A biased version of the same story is contained in Morgenstern (1976).

[12] An anti-neoclassical interpretation of von Neumann's paper founded on mathematical innovations alone, as for example that given by Ingrao e Israel (1987, 200 e 247), does not have sufficient support. In all his scientific life, von Neumann never interested in economics as a science, toward whom his profound and sarcastic skepticism was well known. When he was involved in it, he tried to build models exploiting his revolutionary logical-mathematical insights (see Kuhn and Tucker 1958, Arrow 1989, Dore 1989 and Punzo 1989).

are the shaky methodological foundations, the lack of the so called *lives* variables, the unreal treatment of information and, finally, the static nature of the analysis.

One of Morgenstern's first critical remarks concerned the research methodology of the neoclassical theory and undoubtedly represented one of the principal reasons of his collaboration with von Neumann. In a paper published in 1936, Morgenstern discusses the relationship between logic, as founded by Russell and Whitehead and extended by Hilbert, and social sciences. In the first part of the paper, he points out the hostile attitude of economists towards Hilbert's method. While it seems that other sciences are already converted to an axiomatic approach, the mechanical use in mathematical economics of functional symbolism and numerical techniques produces the repetition of elementary errors, as the adoption of examples and principles transposed mechanically from physics and natural sciences, the abuse of symbols of function without a real utility or, in general, the hastily translation of economic concepts in mathematical expressions.

In a review article on John Hicks' *Value and Capital* published in 1941, Morgenstern goes back on the subject using such caustic remarks that deserve being extensively quoted:

> I refer to the considerable haziness of economic theorizing, to the regrettable fact that concepts are frequently ambiguous, often used in different manners, that their interrelations are not made clear, and, foremost, that the assumptions seldom show a clear relationship either to the facts of everyday life or to those specifically collected and examined. It is especially the latter circumstance which makes the applicability and range of theories uncertain and contributes to the accusation that economic theory is unreal and carried out «in a vacuum». (Morgenstern 1941, 361)

Such a severe criticism does not spare the methodological approach of *Value and Capital*; Hicks' investigation is deemed little rigorous, because it does not offer indications to deepen the relation between economic theory and empirical investigation and, besides, it is a typical example of lack of precision caused by traditional mathematics[13]. Although Hicks states that he has founded a new economic logic, the absence of axiomatic proofs does not allow him to correct his mistakes. In the discussion, for example, of the general equilibrium model, he does not notice a serious error in the proof of the determinateness of the Walrasian system, that is the equivalence between the number of unknowns and the number of equations is neither a necessary

[13] "Instead, the reader will not be so very delighted in view of the wholly unjustified difficulties which the book offers; it is among the most unreadable works that have ever been published on economic theory." (Morgenstern 1941, 364).

nor a sufficient condition for the solution of that model. But such flaws have another more profound motivation, given by the excessive ambition of *Value and Capital*'s project:

> There is, undoubtedly, in the different sciences a point when it is still too early to try to apply methods which later on, when the basis has been more firmly established, may prove to be of greatest importance and consequently yield greater results. (Morgenstern 1941, 375-376)

Not by chance, the same concept is repeated in the first chapter of *Theory of Games*:

> Economists frequently point to much larger, more "burning" questions, and brush everything aside which prevents them from making statements about these. The experience of more advanced sciences, for example physics, indicates that this impatience merely delays progress, including that of the treatment of the "burning" questions. There is no reason to assume the existence of shortcuts. (von Neumann and Morgenstern 1944, 7)

What is at issue here is the option between two different research programmes: is it better to follow the axiomatic method and to obtain, through rigorous procedures, expected or empirically meaningless results, achieving original interpretations only later, or must reality already be studied in its various aspects in the initial phase of the cognitive process? Neoclassical analysis seems to choose the second option. That way, it incurs some criticism, clearly summarized in *Theory of Games*:

a) the formulation of economic problems is so inaccurate that it is impossible to translate them in axiomatic terms;

b) the simpler mathematical tools are used improperly, as for the *mere counting of equations* proposed by Walras;

c) empirical references of economics are confusing;

d) statements are often treated as proofs.

The second part of Morgenstern's criticism is the most important for historical purposes because it concerns the question of the interdependence of economic agents. In his earlier 1928 book and in his 1935 article *Perfect Foresight and Economic Equilibrium*[14], Morgenstern repeatedly claims that the reciprocal influences among economic agents make the principle of neoclassical maximization inadequate. Such method is applicable only to a model of market *à la Robinson Crusoe* (von Neumann

[14] The *Value and Capital*'s review (Morgenstern 1941, 378-379) and the first chapter of *Theory of Games* deal with the same argument.

and Morgenstern 1944, 8), in which the decision maker independently determines the value of all variables. With the rejection of this simplification, a dichotomy emerges. Beside the *dead* variables that depend on the decision maker's will alone, the model now includes the *live* variables as well, in the sense that it also describes the influences of other agents' behavior on the decision maker's will. The final aim of the neoclassical *homo oeconomicus* - the maximization of utility or profit - becomes simply not more available, because in this setting it is determined by the will of every agent included in the model; in its place we find a more complex and qualitatively different interpretative scheme, composed by the solution of various maximization problems conflicting among themselves.

Then, for Morgenstern, it is particularly strange that the Lausanne school has built a theory of the whole economic system ignoring this essential complication; such a paradox has been possible because that theory is founded on "far-reaching restrictions", "restricting devices" or "hidden assumptions" (von Neumann and Morgenstern 1944, 15), as the assumption of perfect competition or the exclusion of the coalitions among agents, that make the Walrasian system worthless from an empirical point of view[15].

These issues are strongly connected with another component of Morgenstern's attack on the neoclassical theory, concerning the unreality of the informative hypotheses. The 1935 paper on economic foresight begins with a critical assessment of the Walrasian general equilibrium model, that is based, implicitly, on the assumption of perfect foresight. Such condition is as essential as empirically unacceptable:

> The impossibly high claims which are attributed to the intellectual efficiency of the economic subject immediately indicate that there are included in this equilibrium system not ordinary men, but rather, at least to one another, exactly equal demi-gods, in case the claim of complete foresight is fulfilled.(Morgenstern 1935a, 173)

But also accepting the abstract nature of the Walrasian system, the problem of its logic consistency remains open. In particular, if a model of competitive economy includes both strategic interaction, with the presence of *live* variables, and perfect foresight, every decision should be the result of an endless chain of conjectural reactions, giving rise to self-evident paradoxes. As an example taken from the literary saga of Sherlock Holmes and quoted by Morgenstern in his 1928 book shows, this sequence of "if-I-think-that-you-think-that-I-think ..." carried *ad infinitum* even by only two agents endowed with perfect foresight prevents them from making any definite resolution. The

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[15] Thus, the Lausanne school avoids "the real difficulty and deals with a verbal problem, which is not the empirically given one."(von Neumann and Morgenstern 1944, 15)

only solution to this indeterminateness is given not by an act of knowledge but by an arbitrary decision which breaks this chain of conjectures. According to this view, it is necessary that economics deals extensively with the limits of the agent's computative and informative powers whereas the Walrasian theory, ignoring them and assuming perfect foresight, creates a science that treats its principal problem as solved[16].

The last part of Morgenstern's criticism of the neoclassical theory is a further effect of his attention to the processing of information. The absence of dynamic considerations in almost all the economic models is a limit so important as to induce Morgenstern to use tones that are not excessive to describe as radical :

> It is clear that a theory of equilibrium which "explains" only a *static situation, which is given as unalterable* and which, because of this basic assumption, is completely unable to say anything about the economy when a variation occurs, is utterly unimportant from a scientific point of view. It would hardly deserve the names of theory and science. (Morgenstern 1935a, 180)

But even if time is implicitly introduced, as in the Walrasian process of price formation, one needs to impose unreal assumptions such as the infinite velocity of agents' reaction of *tâtonnement*. Morgenstern returns to this subject in his article on *Value and Capital*. As everyone knows, Hicks dedicates a large part of his book to create a new dynamic theory, but the reviewer, although he recognizes the great difficulties involved in this attempt, judges it unsuccessful for two reasons: the first is that Hicks defines expectations without introducing either risk or uncertainty; the second regards the vagueness of the concept of plan consistency, from which dynamic equilibrium depends. Both cases raise essential problems that Hicks avoids by tautological statements.

When Morgenstern was writing this last article, he had already begun his collaboration with von Neumann. Therefore he could make an explicit reference to a new method[17] created with the principal aim of overcoming some weaknesses of neoclassical economics. But, as the next section is going to show, game theory is not the only proposal of Morgenstern to achieve this end.

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[16] "Should complete foresight be an indispensable postulate for the erection of the theory of equilibrium, then, there results that wider paradox that the science has already posited the object that it is first to investigate; that, without this assumption, the object could not exist at all in the meaning specifically considered." (Morgenstern 1935a, 175).

[17] "The problems involved are of quite exceptional difficulty and resemble closely those of the theory of games" (Morgenstern 1941, 380).

The proposals of a visionary[18]

So far Morgenstern has been considered an economist who preferred to stress critical tones rather than constructive ones. The principal reason for making this judgement is the influence of the paper published in 1972, in which Morgenstern offered an agenda for future research, listing *thirteen critical points* in contemporary economics. There is little doubt that in his work the efficacy of the proposals did not match the clearness and the validity of the criticism. Such an asymmetric contribution could also explain some faults contained in the *Theory of Games*. Concepts objectively unsuitable for economic theory, as the excessive emphasis on cooperative games, the abuse of minimax theorem or the transformation of non-zero-sum games into zero-sum games through the artifice of the fictitious player, were more an effect of von Neumann's instances of formal perfection than the product of the creative vein, evidently uncertain, of an economist in a crisis of scientific identity like Morgenstern[19]. But in the '30s and '40s he was able to put forward, for every critical judgements just described, as many proposals for changing the foundations of economic theory.

To begin, Morgenstern tries to correct the neoclassical methodological flaws through two proposals: the first is *logistics* and the second is the respect of a *standard of modesty* in economic research.

In the 1936 paper, Morgenstern claims the opportunity to introduce the Hilbertian logic in social sciences, because it allows a meaningful progress as respect Aristotelian and scholastic logic. Being the empirical laws of nature and society fundamentally inexact, only exact methods of reasoning are able to assure strictness and coherence in social sciences. The new logic or *logistics* is a purely formal tool that allows us to recognize with exactitude all the implications of a given set of propositions and to point out the weaknesses of the human mind. Moreover, the axiomatic method permits us to apply functional mathematics to economics correctly, without making the formal mistakes of neoclassical theory, and promotes a process of unification and multidisciplinary integration among scientific languages.

The *standard of modesty* is the method of research followed in the *Theory of Games*. Rejecting the economists' inclination to tackle prematurely *burning questions*, von Neumann and Morgenstern adopt a more lengthened research process:

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[18] "Quite simply, Oskar Morgenstern was a visionary constantly on the lookout for the new and the unusual." (Schotter 1992, 96).

[19] Mirowski (1992, 143-144) points out that these contentions are reflected in some contradictory passages of *Theory of Games*, that have a minor part in its global project anyway. Schmidt (1990) expresses a more critical judgement, attributing to these elements the failure of *economic* game theory in the '50s.

> The sound procedure is to obtain first utmost precision and mastery in a limited field, and then to proceed to another, somewhat wider one, and so on. (von Neumann and Morgenstern 1944, 7)

The foundation of a theory mathematically rigorous and conceptually general requires at least three different stages. In the first stage, in which von Neumann and Morgenstern place their own work[20], the applications have to regard elementary models and sure theories while empirical tests only serve to corroborate the theory. In the following stage, it is possible that economists analyze more complex problems and may produce even not obvious results. But only in the third stage theoretical work can obtain a genuine success conducive to truthful predictions.

On the basis of such an evolutionary pattern, the theory of games needs to reach the second and the third stage to generate original results as well, but in the meanwhile it permits us to represent models with *live* variables. The importance of this last concept for economic analysis was already stressed in the 1928 book, as discussed in the previous section. Therefore Morgenstern was not simply a *facilitator* of von Neumann's work, but he was the principal upholder of the necessity of establishing a strong relation between game theory and economics from their origins. In the review of *Value and Capital*, this issue is only outlined but it is clearly pointed out in the first chapter of the *Theory of Games*. In those pages, game theory becomes the only way to represent two otherwise incompatible elements: a quantitative description and a normative lecture of human acting in strategic interaction conditions. But the final result is not the re-proposal of the maximization principle multiplied by the number of players, but the creation of a fundamentally different framework in which to introduce new concepts of solution.

Although Morgenstern did not offer a direct contribution to this project, he tried to give an original view on a further weakness of economic science. His article on perfect foresight, that criticizes the unreal informative hypotheses of neoclassical theory, contains a model of learning that, even though formally elementary, seems to foresee some characteristics of the more recent literature. The starting point is the idea that an agent may have arbitrary opinions on the correlation between his own behavior and the other agents'. If these opinions reveal themselves wrong, any consequent plan

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[20] "The field covered in this book is very limited, and we approach it in the sense of modesty. We do not worry at all if the results of our study conform with views gained recently or held for a long time, for what is important is the gradual development of a theory, based on a careful analysis of the ordinary everyday interpretation of economic facts. This preliminary stage is necessarily *heuristic*, i.e. the phase of transition from unmathematical plausibility considerations to the formal procedure of mathematics." (von Neumann and Morgenstern 1944, 7).

is doomed with great probability to fail. A failure gives rise to a former opinions' change, that is repeated in the following periods until any improvement in individual welfare is possible. This process of trial and error has the aim of reaching a situation of stability even though nothing guarantees its optimality.

A noticeable feature of this model is the introduction of highly instructed subjects, that are able to evaluate the consequences of their own behavior without being provided with Walrasian perfect foresight. The resulting incomplete knowledge imposes the use of the principles of subjective rationality. So as to clarify this point, Morgenstern makes reference to the concepts of *technical prevedibility* and *effective prevedibility*: the former is founded on the endowment of information common to all agents, the latter depends on the former but it varies from agent to agent according to the level of learning reached. This distinction implies that every agent possesses the same objectively correct model of the world, but that a difference between this last model and the subjective ones also exists.

In an article on Morgenstern's scientific heritage (Schotter 1992), the author ascribes to this model the merit of having anticipated the rational expectation common-model solution, but at the same time he argues that Morgenstern would have rejected it:

> My feeling is that Morgenstern would have been more inclined to think of the agents in the world as adhering simultaneously to many theories and to think, in truly Austrian fashion, that many subjectively correct models of the real world exist, reality being determined, in part, by the different subjective models that people use. A sunspot model might even be closer to the type of analysis he might have envisioned. Hence, while in some sense the theory of rational expectation equilibrium would have been a very welcome event for Morgenstern since it dealt with precisely the problem that first aroused his interest in game theory, its treatment in the profession might ultimately have left him dissatisfied. (Schotter 1992, p. 110)

Finally, Morgenstern contributed to the foundation of a dynamic theory identifying three different lines of research. One is an outgrowth of game theory for it regards the dynamic character of the concept of strategy. In a passage quoted from a 1949 *Kyklos* paper, Morgenstern states,

> A strategy envisages successive moves made in response to expected moves and countermoves of the opponent, so that the description of a play already takes cognizance of its extension over time. For that reason it is not *a priori* clear what is meant when a "dynamic" theory of games is demanded. (Morgenstern 1949, 307)

A second tentative proposal to describe dynamically a static economic concept is contained in Morgenstern (1948) and provides an original description of the reconstituted demand function introduced by Allen (1938) and developed later by Shubik (1959a). The third suggestion is the object of a 1935 paper concerning the relations between time and value theory. However, these two last contributions have serious limitations in respect of the solution of the problems which they deal with. The demand function does not become a useful tool of analysis, while the scheme presented in *The Time Moment in Value Theory* is a quite vague attempt to classify the temporal relations between income and consumption. In both cases, Morgenstern's conclusions recognize the temporary value of his proposals postponing a real progress on the same subjects to subsequent research.

In the light of this discussion, Morgenstern appears an author more inclined to be a critic of the standard economic theory than a promoter of new ideas, but this judgement does not reduce the importance of his principal contribution, the connection between the application of game theory to economics and the abandonment of the orthodox conception of economic behavior.

The rebirth of the heterodox vocation

In a recent attempt to analyze the state of the discipline, Robert Aumann (1985) defines game theory as a logical-mathematical tool employed to elaborate a normative theory of rational behavior in conditions of strategic interaction. He takes the view that the economic applications of game theory are characterized by a conception of *strong* rationality. So as to clarify such notion, Aumann discusses the difference between *homo rationalis* and *homo sapiens*; the first, a mythical figure, would represent, for a long process of historical accumulation, the fittest tool to formally describe some aspects of the behavior of *homo sapiens*, that is the real one. The theory of games, as a normative discipline, confines itself to deal with *homo rationalis* and would have descriptive power only as far as *homo sapiens* coincides with *homo rationalis*. Being, in Aumann's view, the rationality principle and the utility maximization nearly identical, it ensues that game theory has descriptive powers only if *homo sapiens* is a maximizer.

But both the identity between rationality and maximization and the outright acceptance of the maximization principle as a description of the behavior of *homo sapiens* are anything but evident allegations according to Morgenstern's view. If interpreted correctly, his attack to the neoclassical system is indeed directed more to its whole structure than to single elements. Abstracting from the methodological matter,

Morgenstern's arguments defend a descriptive conception of economic theory against a conception of *strong* rationality as that described by Aumann. Moreover he shows that besides weakening the neoclassical building, the application of game theory is incompatible with his mainstay, that is the principle of constrained maximization.

This statement undoubtedly characterized the *Theory of Games* as well. In its first pages, the authors declare that they want to show how economic behavior "is in many respects quite different from the way in which they are conceived at the present time" and to propose solutions that "diverge considerably from the techniques applied by older or by contemporary mathematical economists" (von Neumann e Morgenstern 1944, 1). After a few pages, they deal again with the same point, stating that in presence of at least two economic agents,

> each participant attempts to maximize a function (his above-mentioned "result") of which he does not control all variables. This is certainly no maximum problems, but a peculiar and disconcerting mixture of several conflicting maximum problems. Every participant is guided by another principle and neither determines all variables which affect his interest.
>
> This kind of problem is nowhere dealt with in classical mathematics. We emphasize at the risk of being pedantic that this is no conditional maximum problem, no problem of the calculus of variations, of functional analysis, etc. It arises in full clarity, even in the most "elementary" situations, e.g., when all variables can assume only a finite number of values. (von Neumann and Morgenstern 1944, 11)

In this quotation, it is easy to see Morgenstern's influence, even if the necessity of giving up the maximization principle inspires many other proposals of that book, as the use of mixed strategy to represent the *bluff* or the *arbitrary act* invoked in Morgenstern (1935a, 174), the introduction of coalitions or the proposal of *minimax* as a unique method of solution for all kinds of game[21].

Then, it is not surprising that Morgenstern confirmed the same view many years later. In the article published in 1972, "Thirteen Critical Points", the first critical remark concerns the inadequacy of the principle of constrained maximization for a social science such as economics, even if thirty years after *Theory of Games*, as Morgenstern himself writes, Samuelson still judges it a basic tenet in his 1972 Nobel lecture.

Therefore, the introduction of game theory into economics for its first upholder is a successful attempt to reject the principle of neoclassical maximization in favour of qualitatively different tools, as those constituting recent game theory. But this view clashes, for example, with Aumann's position:

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[21] The stress on *minimax* is attributable to von Neumann, who had already proposed it in his 1928 paper to find a determined solution to the *games of strategy*, and not to Morgenstern, who accepted openly the principle of indeterminacy in social sciences (see Schotter 1992, 107-108).

> The Nash equilibrium is the embodiment of the idea that economic agents are rational; that they simultaneously act to maximize their utility. (Aumann 1985, 43)

Such a strict logical connection between game theoretical rationality and an orthodox conception of economic rationality is quite simply non-existent. Only in the latter, and insofar as it is permitted by the rigid neoclassical hypotheses, it is possible to define a maximizing behavior as rational. In the former, as it can be interpreted on the basis of the Nash equilibrium, the decisive condition to define a choice as rational is that any player believes that other players do not have a reason to make different choices from those provided in equilibrium: the simultaneity concerns the reciprocal relevance of the player's choices, not the utility maximization that is an objective subordinate to the first one. Within game theory, a simple and direct translation of the rationality principle does not exist and a normative theory concerning strategic models is unavoidably the outcome of a mixture of perceptions, conjectures and reasoning, whose representation is absent from the orthodox economic models.

In a 1991 *Econometrica* paper, Arthur Rubinstein stresses the importance of these remarks to a descriptive interpretation of the relation between economics and game theory, but in his conclusions he vaguely claims a more vivid psychological colouring of game theory[22]. A more concrete view is expressed by one of the discussants of Aumann's paper, Reinhard Selten, who writes:

> Aumann's arguments sound like a defense of present research practices based on unquestioned strong rationality assumptions. Even if this kind of theorizing may have to go on for some time in the absence of superior alternatives, a redirection of efforts towards the development of a descriptive theory of game behavior seems to be desirable. (Selten 1985, 77)

So as to criticize Aumann's *sophisticated* rationalism, which is different from the *naive* rationalism of those who regard the maximization principle as self-evident, Selten refers to the more recent experimental literature, whose results weaken the empirical plausibility of the principle and impose the development of a descriptive game theory. Selten's own contribution in this direction is well known, and has been documented as *complete theories* and refinements of Nash equilibrium in the handbooks of game theory. But, replying to Aumann, Selten mentions other contributions, such as Nelson

[22] "Thus, if a game in the formal sense has any coherent interpretation, it has to be understood to include explicit data on the player's reasoning processes. Alternatively, we should add more detail to the description of these reasoning procedures. We are attracted to game theory because it deals with the mind. Incorporating psychological elements which distinguish our minds from machines will make game theory even more exciting and certainly more meaningful." (Rubinstein 1991, 923).

and Winter' s evolutionary model (1982) and Tietz and Weber's limited rationality (1972).

In addition to these references, it is possible to add other examples of economic applications of game theory that weaken or remove the principle of maximization.

A first instance is represented by the various behavioral theories derived from experimental economics, a discipline that besides its foundation also draws its principal interpretative schemes from game theory[23].

While this relation appears mainly formal, more substantive examples of heterodox applications of game theory belong to the research area founded on variations of the classical Prisoner's Dilemma (Campbell and Sowden 1985). A part of these works deals with forms of limited rationality and assumes, for example, that deviations provoking slight gains can be ignored (Radner 1986) or that players are provided with *bounded recall* (Lehrer 1988). A different direction of research finds a meaningful outcome in the proof of the strategic importance of reputation, given by Kreps, Milgrom, Roberts and Wilson (1982). The presence of altruistic motivation is also a consequence of dynamic considerations in welfare economics, as the discussion of Prisoner's Dilemma in Sen (1970) and in Axelrod (1984) explains. By applying repeated games and *folk theorem*, this literature provides itself with means to formally support the cooperation between economic agents.

But the most interesting research area for present purpose is learning theory. An insight, drawn by Kreps (1990, 169-182), gives an intuitive understanding of the importance of game theory to this field. A failing of most learning models that do not use game theory is to consider only competitive situations and sequences of temporary equilibria. In that way, the agents' behavior must be derived deductively in order to respect this interpretative framework. On the contrary, game theory allows adopting the inverse process of reasoning of fixing both inductively and deductively the assumptions of behavior before and then modelling the learning procedures. Moreover, game theory permits us to describe disequilibrium situations and to explicitly include the mutual influences among economic agents. Such a greater generality of application explains the recent development of game theoretic learning models, which are going to replace the literature on learning in rational expectations models[24].

A common hypothesis present in learning models is to include myopic players, that maximize within every single period. This reference allows this short list to end mentioning the attempts to originally describe dynamic inconsistency. Such problem, discussed firstly by Strotz (1955-56), has been studied by means of game theory into

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[23] For an account of the origins of experimental economics, see Innocenti (1994).

[24] Besides the last chapter of Kreps (1990), a review of learning strategic models is contained in Battigalli, Gilli and Molinari (1992). For the comparison with the rational expectations models, the reference is Sargent (1993).

economic politics by Kydland and Prescott (1977), but the utility of such application had already been stressed by Phelps and Pollak (1968), Peleg and Yaari (1973) and Hammond (1976). So as to classify these contributions[25], it is useful to resort to an analytical fiction: a dynamically incoherent behavior originates because an agent behaves *as if* he is composed of more selves that alternatively determine intertemporal decisions. On this account, a process of individual choice could be analyzed as a situation of strategic interaction. A further consequence is that the solutions of the dynamic incoherence problem could correspond to different criteria for ordering the successive selves of the same agent. This approach allows to tackle a variety of psychological problems ignored by the orthodox economic conception of rationality, as self-commitment, wishful thinking or *akrasia*[26].

These brief references support the view that the theme of imperfect rationality - which may be limited, procedural, quasi-rationality and so on - is a fertile ground for improving the contribution of game theory to economics and for reaffirming a fundamental component of Morgenstern's original project.

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[25] For an original analysis and some bibliographical references, see McClennen (1990).

[26] A wide discussion of such problems appears in Elster (1985).

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