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Linking Strategic Interaction and Bargaining Theory.
The Harsanyi - Schelling Debate on the Axiom of Symmetry

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Abstract. This paper analyses the early contributions of John Harsanyi and Thomas C. Schelling to bargaining theory. In his work, Harsanyi (1956) draws Nash's solution to two-person cooperative games from the bargaining model proposed by Zeuthen (1930). Whereas Schelling (1960) proposes a multi-faceted theory of conflict that, without dismissing the assumption of rational behaviour, points out some of its paradoxical consequences. Harsanyi and Schelling's contrasting views on the axiom of symmetry, as postulated by Nash (1950), are then presented. The analysis of this debate illustrates that, although in the early 1960s two different approaches to link strategic interaction and bargaining theory were proposed, only Harsanyi's insights were fully developed later. Lastly, the causes of this evolution are assessed.

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1. Introduction

Of Nash's two contributions to game theory, it is the concept of the equilibrium point that has been extensively applied to economics. This can be attributed to the fact that this concept exemplifies the dominant view among economists that the maximization postulate and the assumptions of perfect rationality and complete information have both normative and descriptive value. This reasoning however is not so readily applicable when it comes to the bargaining model proposed by Nash in the early 1950s. One discrepancy is that, for instance, the removal of the time aspect is a feature of the model which greatly differs from the standard economic approach to bargaining. Moreover, and more generally, the set of axioms proposed by Nash to determine a solution to the bargaining game represented a major departure from the more commonly used price-theoretical analysis. In spite of these divergences, Stahl (1972) and Rubinstein (1982) developed the Nash program, and it is their extensions that represent the dominant approach in strategic bargaining theory today. The course that this evolution has taken raises at least two questions. First, how did Nash's model gain popularity among economists? Second, why did the axiomatic approach displace alternative and concurrent ways of modelling bargaining processes?

These questions become even more compelling if we take into account the fact that bargaining theory is not presently considered a flourishing research area. This might be attributable to the widespread belief that the dominant approach to strategic bargaining is too stylized to capture the richness of the real world (Fudenberg and Tirole 1991). It has been argued that focusing on the *fundamentals* (Muthoo 1999) is usually justified by the need to solve the simpler problems before seeking to tackle the more difficult ones (Binmore and Dasgupta 1987). Indeed, the weaknesses of strategic bargaining models reveal how game theory often has little interpretative power when applied to empirically relevant matters (Kreps 1990).

To provide some insight into these issues this paper analyses the early contributions of John Harsanyi and Thomas C. Schelling. Although both scholars contributed to the foundations of the theory of strategic bargaining, their work was characterized by two very different methodological approaches. While Harsanyi aimed at selecting single solutions, developing deductive models, and finding conditions for static equilibrium, Schelling allowed for multiple outcomes, relied on intuition and empirical counterarguments, and made the dynamic features of the negotiation process explicit. A specific episode from the 1960s will help to understand the contrast in the work of the two. Criticisms addressed to Nash's axiomatic model by many reviewers had focused on the assumption of symmetry that was judged more appropriate prescriptively than descriptively. Harsanyi and Schelling discussed the matter in two papers that point out their contrasting views on the methodology of economics.[1] The debate shows how in the early 1960s, when game theory had not yet entered the economists' tool-box, two distinct methods for linking strategic interaction and bargaining theory were proposed, although they were differently acknowledged later. Harsanyi actively contributed to defining a research program which, subsequently, was to become the dominant

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[1] Harsanyi and Schelling did not discuss this issue directly. Schelling gives this account of his interaction with Harsanyi: "Harsanyi and I overlapped at Yale University for about two years in the 1950s but we had little communication. That was partly because he was shy and reticent, partly I think because his English was not comfortable for him. I believe others had the same experience. I published an article, *On the Abandonment of Symmetry in Game Theory* in August, 1959, eighteen months after I had left Yale; I included it in my book, The Strategy of Conflict, 1960. It mainly targeted Harsanyi's work, but I am not aware that he paid any attention to it. We certainly did not correspond about it. I wish we had. That's about all I can tell you. I wish I had more." (Schelling 2005)

program, while Schelling's insights were further developed more by other social scientists than by economists. In order to better understand the course of these developments, it is useful to describe the nature of the Harsanyi-Schelling disagreement on the assumption of symmetry and to discuss how it relates to subsequent developments.

Section 2 below presents Harsanyi's (1956) proof of the equivalence between Nash's and Zeuthen's bargaining models. Section 3 outlines the main characteristics of Schelling's theory of conflict. The distinct methodological approaches of Harsanyi and Schelling are analysed in section 4 by discussing their contrasting views on the axiom of symmetry. Conclusions are drawn in the final section.

2. Harsanyi as the interpreter of Nash's bargaining model

The main legacy of Harsanyi regarding the application of game theory to economics is the proof that any kind of uncertainty can be modelled in a Bayesian game (Harsanyi 1967-68). Prior to that, Harsanyi (1956) had already played the role of the interpreter of a game theoretical model for the wide audience of economists by demonstrating that Nash's bargaining model was mathematically equivalent to the model proposed earlier by the economist Frederik Zeuthen (1930). The immediate consequence was that Nash's contribution, albeit expressed in a new formal language, was finally accepted as fully integrated in the body of economics.

It is well known that Nash's interest in economics was quite incidental (Leonard 1994, p. 497). His only training in economic theory was an undergraduate course in international economics attended in 1948, which inspired his first paper on bargaining theory (Nash 1950). Thus, it is not surprising that the publications granting him the Nobel Prize are almost entirely devoid of economic references. His two papers founding strategic bargaining theory (Nash 1950, 1953) make no exception. This lack of empirical references is explained very simply by the first lines of *The Bargaining Problem*: "In order to give a theoretical treatment of bargaining situations we abstract from the situation to form a mathematical model in terms of which to develop the theory." (Nash 1950, p. 156). Basically, what Nash expunged from his model was exactly the core of the bargaining problem, namely, the process of negotiation through which an agreement is eventually reached.[2] Harsanyi's (1956) proof intended to show that the reintroduction of the negotiation procedure, such as the one postulated by Zeuthen, did not modify the bargaining solution determined axiomatically by Nash.[3] The consequence

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[2] Binmore and Dasgupta single out this characteristic of Nash's model as essential: "Within the Nash program, each of these and other cooperative solution concepts are seen as an attempt to describe the outcome of a non-cooperative negotiation game *without formalizing the negotiation procedure.*" (Binmore and Dasgupta 1987, p. 8)

[3] "Many economists as well as other empirical social scientists will probably find Zeuthen's reasoning more convincing as it is based on a fairly plausible psychological model of the bargaining process, and, at the same time, they may look somewhat askance at Nash's game-theoretical method, which relies on abstract mathematical postulates whose empirical relevance may be less obvious. But, in actual fact, the two approaches are complementary. The abstract axiomatic method should primarily be regarded as a heuristic device. If it can be shown that there is, consistent with people's behaviour in a certain situation, one and only one hypothesis satisfying certain attractive general postulates (whether formal postulates based on considerations of mathematical simplicity, continuity, symmetry, determinateness, etc., or *material* postulates suggested by observation or introspection), this is a strong argument *for* this particular hypothesis – or at least *against* any alternative ones – even if no convincing psychological model is known which would explain the behaviour predicted by this hypothesis. Of course, if a satisfactory psychological model is also found, this will further increase our

of this was that the adoption of a new formal method could more concisely produce the same outcome given by traditional price theory. Before Nash, bargaining was usually modelled in terms of successive steps mimicking the price adjustment process. This approach, dating back to Edgeworth (1881) and developed by Zeuthen (1930), Hicks (1932), and Pigou (1932) was ignored by Nash who solved the bargaining problem by means of two different and complementary methods. The first was the axiomatic definition of bargainers' rational expectations, and the second was the reduction of the bargaining cooperative game to a non-cooperative game that was solvable by means of equilibrium points. Harsanyi (1956) named the first approach *the theory of bargaining with given threats* and the second *the theory of optimal threats*, but contrary to what was to become the standard view he assumed that the latter followed out from the former, and this was the object of his proof as described below.

Zeuthen (1930) determines a solution to the bargaining process by postulating a sequence of concessions made by the bargainer who is less willing to incur the risk of a disagreement. If bargainers 1 and 2 respectively demand a_1 and a_2, which give utilities $u_1(a_1) > u_1(a_2)$ and $u_2(a_2) < u_2(a_1)$, and the bargainers assess p_1 and p_2 as the probabilities that the opponent will reject the respective demands, then bargainer 1 will find it advantageous to insist on demand a_1 only if

$$(1) \qquad [u_1(a_1) - u_1(a_2)] / u_1(a_1) > p_1$$

Similarly, bargainer 2 will insist on demand a_2 if

$$(2) \qquad [u_2(a_2) - u_2(a_1)] / u_2(a_2) > p_2$$

If these two ratios are known by both bargainers, it can be imposed that bargainer 1 will lower his demand a_1 if

$$(3) \qquad [u_1(a_1) - u_1(a_2)] / u_1(a_1) < [u_2(a_2) - u_2(a_1)] / u_2(a_2)$$

Otherwise bargainer 2 lowers his demand a_2.[4] Lastly, Zeuthen assumes that each concession is large enough to reverse the inequality sign and hence to determine a concession by the other bargainer. In this way the agreement is determined provided that demands are measured in indivisible units.

Harsanyi (1956) first shows algebraically that Zeuthen's solution is the maximized product of the utilities gained by the bargainers, which is exactly what Nash's solution would result in. By algebraic manipulation the inequality (3) can be written:

$$(4) \qquad u_1(a_1) \cdot u_2(a_1) < u_1(a_2) \cdot u_2(a_2)$$

Consequently, it can be derived that the sequence of concessions stops when the product of the utilities associated to the agreement is maximized.

After this, Harsanyi derives axiomatically the key assumption of Zeuthen's model: the bargainer who is less willing to risk a conflict is the bargainer who will make a concession.

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confidence in our hypothesis. But the converse is equally true. Our confidence in a given psychological model will certainly increase if the pattern of behaviour predicted by it is found to be one and the only one compatible with certain general postulates that are otherwise attractive to us." (Harsanyi 1956, p. 151)

[4] If the two ratios are equal, the model assumes that both bargainers lower their demands.

The proof is based on four axioms: perfect knowledge, monotonity of the probabilities p_1 and p_2 with respect to the bargainers' utility gains, expected-utility maximization, and symmetry.

In light of its methodological implications it is the last axiom that merits attention: "The bargaining parties will follow identical (symmetric) rules of behaviour (whether because they follow the same principles of rational behaviour or because they are subject to the same psychological laws)." This definition is so glossed in the footnote: "This postulate is here equivalent to the assumption which we would express on the common-sense level by saying that each party will make a concession at a given stage of the negotiations if and only if he thinks he has at least as much *reason* as his opponent has to yield ground at that point." (Harsanyi 1956, p. 149)

The word reason in italics would ask for further explanation. It is indeed evident that the axiom involves a key methodological issue that Harsanyi emphasises by claiming that Nash's theory with given threats is just a *generalization* of the principle of symmetry.[5] Not surprisingly, Harsanyi comes back again to this point in the following years with arguments that deserve to be examined closely. Before doing this though, it is useful to take a step back to Nash's original contribution.

The axiom of symmetry was introduced by Nash in his first paper and was deemed as a necessary condition to determine the solution of the bargaining game. Nash's comment on the axiom is laconic: it "expresses equality of bargaining skill" (Nash 1950, p. 159). Nash (1953) extends and modifies this definition: "The symmetry axiom, Axiom IV, says that the only significant (in determining the value of the game) differences between the players are those which are included in the mathematical description of the game, which includes their different sets of strategies and utility functions. One may think of Axiom IV as requiring the players to be intelligent and rational beings. But we think it is a mistake to regard this as expressing *equal bargaining ability* of the players, in spite of a statement to this effect in *The Bargaining Problem*. With people who are sufficiently intelligent and rational there should not be any question of *bargaining ability*, a term which suggests something like skill in duping the other fellow. The usual haggling process is based on imperfect information, the hagglers trying to propagandize each other into misconceptions of the utilities involved. Our assumption of complete information makes such an attempt meaningless." (Nash 1953, pp. 137-8)

In this way, Nash explicitly assumes that besides being perfectly informed players have to be equally *intelligent and rational* to find an agreement. This condition has two consequences. First, it enables one to select a unique outcome that would otherwise be indeterminate by means of the mere mathematical description of the game. Second, the outcome of the bargaining game ceases to depend on the negotiation process and becomes the necessary result of the initial comparison of demands as captured by the maximization of the utility functions.

Harsanyi (1956) strictly adopts Nash's later interpretation,[6] but only five years later newly addresses the point in a paper on cooperative game theory. In this context the axiom of

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[5] In a footnote Harsanyi adds the following remark: "The present writer feels that this aspect of Nash's approach deserved greater emphasis than it received in Nash's original exposition." (Harsanyi 1956, p. 149)

[6] "The outcome may depend significantly on such *accidental* factors as the bargaining skill of the two parties' actual representatives. But it is a perfectly legitimate question (and one to which we are entitled to an answer from economics and political science) to ask what sort of agreement tends to result from a given objective economic and political balance of power between two parties (as well as from given subjective attitudes on their part) if all disturbing forces are assumed away (e.g., by postulating perfect rationality and perfect knowledge)" (Harsanyi 1956, p. 145).

symmetry assumes a slightly different meaning: "Postulate 4 [symmetry] is also based on the principle that the two players expect each other to act rationally. Its purpose is to exclude bargaining strategies that a given player could adopt only on the expectation that the other player would follow a different, and less rational, bargaining strategy. In particular, it wants to rule out that a player should act on the expectation that a rational opponent would make a concession in a situation in which he himself, following his own criteria of rational behavior, would refuse to make a concession." (Harsanyi 1961, p. 185)

This is the equivalent of saying that a solution to the bargaining game (i.e. the equilibrium) can be defined only if each player's expectations of other player's behaviour are correct. This result however depends on two conditions. The first condition is that those expectations can be correctly assessed only if each player conjectures that the counterpart thinks exactly like himself. It is the definition of rationality which guarantees this outcome in that it selects maximizing behaviour and excludes any alternate kind of behaviour. The second condition is even harder to meet. Players are assumed to be identical for all the relevant variables of the model: "As any theory must apply to *both* players, if the two players happen to be equal with respect to all relevant *independent* variables they must be assigned full equality also with respect to the *dependent* variables, i.e., with respect to the outcome. But this is precisely what the symmetry postulate says. Different theories of bargaining may differ in what variables they regard as the *relevant* independent variables – but, if the two players are equal on *all* variables regarded by the theory as relevant, the theory must allot both players the same payoffs." (Harsanyi 1961, p. 189)

In the same paper the meaning of this revised interpretation is emphasized by a modified proof of the equivalence between Zeuthen's and Nash's models. Two more postulates are added, although they appear redundant. The fifth axiom, which is named *Restriction of variables*, imposes that bargainers' decisions are dependent only on the variables implied by the other axioms. In Harsanyi's words, "Postulate 5 serves to exclude, from the players' decision rules, irrelevant variables extraneous to our rationality postulates. If we dropped postulate 5, our remaining five postulates would not rule out some quite arbitrary, or even silly, decision rules. For instance, it would be quite compatible with our other five postulates that the players should divide any joint profit in proportion to their telephone numbers, or in proportion to the logarithms of their waist measurements, etc." (Harsanyi 1961, p. 185)[7]

Harsanyi's subtly ironical vein is revealing. The introduction of a specific postulate whose sole purpose is to restate the others cannot be justified by mere formal arguments.[8] The main reason for inflating the model of an unnecessary axiom can be found in the debate raised

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[7] Harsanyi (1966) restates the axiom in the following way: "The purpose of this postulate is to exclude some completely arbitrary decision rules, such as, e. g., making the players' payoffs proportional to their telephone numbers, etc. Many of these arbitrary decision rules would be quite consistent with all our other postulates. Our last postulate, however, rules them out on the ground that there is no reason to regard, e. g., telephone numbers as *relevant* variables in deciding the players' payoffs, etc. More generally, the postulate implies that the *only* variables influencing the players' bargaining behavior will be: (i) the *payoffs* associated with alternative outcomes for each of the players, and (ii) the *subjective probabilities* each player assigns to different possible outcomes being accepted or rejected by the other player(s). Among these variables, only those mentioned under (i) are *independent* variables while the variables under (ii) are themselves determined by the variables under (i)." (Harsanyi 1966, p. 621)

[8] The other new axiom (the sixth) of *mutually expected rationality* is also formally superfluous because it imposes "that players expect each other to follow decision rules consistent with the present postulates." (Harsanyi 1961, p. 184)

within the scientific community by Nash's bargaining model. Significantly, Harsanyi (1961) stresses its importance by claiming that in the 1956 proof it was subsumed within the assumption of symmetry,[9] but the decision to keep it separate is the result of its being "the really crucial assumption. This is because our model differs mainly from alternative theories of bargaining, such as Raiffa's, Braithwaite's, and Schelling's, in making the two parties' bargaining strategies dependent only on a restricted set of variables which are directly relevant for the rational pursuit of individual utility by the two parties – whereas Raiffa and Braithwaite make the two parties' strategies dependent on certain ethical criteria, and Schelling makes them dependent on certain psychological considerations. In our model these ethical and psychological variables are excluded by the restriction-of-variables postulate." (Harsanyi 1961, p. 188)

Thus, the postulate of restriction of variable serves to emphasize that the Zeuthen-Nash model is different not only from Raiffa's and Braithwaite's arbitration schemes but also from the theory of conflict proposed by Schelling, to which Harsanyi attributed the introduction of psychological variables in bargaining theory. To better explain what Harsanyi intended to distance himself from, it is opportune to outline what the contribution of Schelling consists of.

3. Schelling's empiricism as the basis for a theory of conflict

A common view in the historical assessment of Schelling's contribution to economics is that, notwithstanding the recent attribution of the Nobel prize, his insights have not been fully developed yet.[10] One reason for this was highlighted by two earlier reviews of Schelling's first book written by Robert Bishop and Martin Shubik and respectively published in the *American Economic Review* and in *The Journal of Political Economy*. In their first remarks both reviewers addressed the fact that the content of the book was hard to classify. In regard to Schelling's informal use of game theory Bishop first comments: "Game theory is too important and too rich a topic to be left exclusively to the mathematicians. Though never stated quite so bluntly, this seems to be the central unifying theme of the present collection of essays." (Bishop 1961, p. 674). Shubik, on the other hand, denied that any relation existed between the book and game theory: "It is my opinion that this book would have been a much stronger contribution had most of the references to game theory been deleted. Although the formal structure of that topic could have been of considerable assistance to the type of analysis presented by Schelling, there is little evidence that it has been used." (Shubik 1961, p. 502) Both reviewers subsequently went on to argue that the interdisciplinary nature of the book was such that economists ascribed it to the field of game theorists and game theorists to the field of economists. Ultimately, this aspect was judged critically by Shubik: "Its weakness lies in the author's apparent lack of appreciation of the power of the methodology which has already been developed and which could have been of considerable use in furthering his own analysis." (Shubik 1961, p. 502)

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[9] "In my earlier paper I did not introduce the restriction-of-variables postulate as a separate assumption distinct from the symmetry postulate. But now I think that the logical structure of our model comes out more clearly if these two assumptions are made separate postulates." (Harsanyi 1961, p. 188)

[10] Kreps (1990) claims that although Schelling's treatment of the criteria for selecting multiple equilibria remains the most stimulating discussion of this issue in game theory, "little to no progress has been made in exploring Schelling's insights" (p. 101). See also Crawford (1991), Binmore, Osborne, and Rubinstein (1992), Myerson (2001), Sugden (2001).

Judgments such as these point out that Schelling was immediately perceived as an outsider. His intellectual originality and his inclination to an interdisciplinary view were far from the common way of thinking among economists at the time. His theory of conflict made no exception, in spite of its being far-sighted in its methodological foundations: "we seriously restrict ourselves by the assumption of rational behavior – not just of intelligent behavior, but of behavior motivated by a conscious calculation of advantages, a calculation that in turn is based on an explicit and internally consistent value system. (…) The advantage of cultivating the area of *strategy* for theoretical development is not that, of all possible approaches, it is the one that evidently stays closest to the truth, but that the assumption of rational behavior is a productive one. It gives a grip on the subject that is peculiarly conducive to the development of theory." (Schelling 1960, p. 4)

This starting point of Schelling's was consistent with the yet to come rational approach to economics, although he believed that such foundations had to be taken with all due caution: "We thus limit the applicability of any results we reach. If our interest is the study of actual behavior, the results we reach under this constraint may prove to be either a good approximation of reality or a caricature. Any abstraction runs a risk of this sort, and we have to be prepared to use judgment with any results we reach."(Schelling 1960, p. 4)

His attitude was based on the awareness that "it is not a universal advantage in situations of conflict to be inalienably and manifestly rational in decision and motivation. Many of the attributes of rationality, as in several illustrations mentioned earlier, are strategic disabilities in certain conflict situations. It may be perfectly rational to wish oneself not altogether rational, or – if that language is philosophically objectionable – to wish for the power to suspend certain rational capabilities in particular situations." (Schelling 1960, p. 18)

The consequences of this methodological view could not but be unconventional. A non-exhaustive list includes the proposal of focal points as a solution to the equilibrium selection problem, the proof of the rationality of irrevocable commitments, and the foundation of bargaining theory as an empirical and experimental-based theory.

One motif that runs through the whole of Schelling's book is that of how one person can make another believe something. Indeed, even when dealing with the problem of equilibrium selection he seeks to provide an answer to this question. As Schelling puts it, game theory is empirically significant if it takes into account *real* strategic environments that are characterized by two kinds of interdependency: the first is that each player's best action depends on what he expects other players to do, the second is that each player tries to influence the other players' choices by trying to influence their expectations of his choices. If the latter kind of interdependency is taken into account, players coordinate their expectations by exploiting all the clues made available by playing the game. Focal points are therefore the proposal of the concepts of shared prominence or salience for that purpose. If, by definition, focal points are strictly and irrevocably contingent on the actual conditions in which the game is played, it follows that not having a systematic arrangement of the focal point determinants is not a flaw but a necessary requisite of Schelling's theory: "People *can* often concert their intentions or expectations with others if each knows the other is trying to do the same. Most situations – perhaps every situation for people who are practiced at this kind of game – provide some clue for coordinating behavior, some focal point for each person's expectation of what the other expects him to expect to be expected to do. Finding the key, or rather finding *a* key – any key that is mutually recognized as the key becomes *the* key – may depend on imagination more than on logic; it may depend on analogy, precedent, accidental arrangement, symmetry, aesthetic or geometric configuration, casuistic reasoning, and who the parties are and what they know about each other." (Schelling 1960, p. 57) If we accept that imagination matters more than logic in sharing beliefs, the determination of focal points

becomes essentially an intuitive process which thus merits an empirical rather than analytical approach. The factors of expectations convergence listed in the above quotation are in fact endogenously determined which implies that insight into the equilibrium selection process can be provided only by the reference to empirical examples. Schelling, in his book, discusses a long series of cases to draw the conclusion that there are no regularities at all. Nevertheless this perspective enables him to select a paradoxical consequence. Namely, implicit coordination may work better than explicit coordination since more information or communication may be disadvantageous if it detracts from the intrinsic magnetism of determinate outcomes.

Schelling uses a similar reasoning to prove the rationality of irrevocable commitments that represents another empirically based paradox. According to Schelling, observation often shows that bargainers voluntarily and irreversibly sacrifice freedom of choice. He deems that this is the result of the fact that the power to constrain the other bargainer may depend on the possibility of binding oneself. Thus, the paradoxical consequence is that a lack of restrictions may become a source of weakness while absence of freedom may imply strength.

Schelling then applies this interpretation to situations of international negotiations. He points out how, in actual bargaining processes, one side can accept irrevocable commitments in a way that is patently visible to the other side. This tactic - which in the case of governments may depend on a binding public opinion - has the effect of squeezing the range of indeterminacy down to the point most favorable to the constrained party. Basically, this occurs because an action which increases the future cost of backing down from one's demand may effectively convince the other part that one is not in fact willing to retreat. This observation is subsequently extended by Schelling in an investigation of the institutional and structural characteristics that may influence the efficacy of the commitment.

This last point is strictly related to the discussion of threats as tools of commitment. Nash (1953) includes threats in his model as the first step of the negotiation process. According to Nash, the final outcome is the result of a two moves game. In the first move players inform each other of their threats, which are the payoffs if no agreement is reached. In the second move they inform each other of their demands. By applying the optimization postulate, the status quo established by the threat strategies is the point upon which the ultimate payoff is determined. In contrast with Nash's model, Schelling contends that threats have an essential psychological component in that it is not the content which matters most but the capacity of the threat to impress the other party. Indeed, the threatening party has nothing to gain by carrying out their threat, yet must convince the threatened party of the contrary. According to Schelling solutions to this problem are provided by assuming commitments which have blatant credibility. As is often the case with promises, threats are made in an effort to change the other person's mind by manipulating their expectations of how you, in turn, will behave.

The deep psychological foundations of Schelling's paradox may explain its lack of applications in economics. This approach was deemed more applicable in other social sciences than in economics. Putnam (1988), for instance, introduces the principle of irrevocable commitments under the label of *paradox of weakness* in a model of negotiation between countries, using the principle to successfully demonstrate that smaller countries are more adept at exploiting their constraints to advance their own interests than larger countries.

As a final point, Schelling also contributes to laying the foundation of the strategic theory of bargaining as an empirical and experimental based theory. His main motivation in doing so is his belief that deductive arguments cannot the whole work make. Schelling reasons that since the ultimate outcome of the bargaining is inherently linked to the negotiation process, it is the amount of contextual details included in the analysis that ups the

solvability of a bargaining game. Schelling uses a two-player non-zero-sum game to discuss this point. He illustrates that if there are two players and no pure opposition of interests, their behavior cannot stem from a priori considerations as neither introspection nor axiomatic reasoning can successfully perform this task. Thus, in this case, the decisions of two interacting *centers of consciousness* cannot be predicted without relying on empirical evidence. In a further provocative example, Schelling asks whether game theory could help two individuals looking at the same ink blot and trying to identify the same picture in it: "The answer to this question can be found only by trying. But, if they can, they can do something that no *purely formal* game theory can take into account; they can do *better* than a purely deductive game theory would predict." (Schelling 1960, p. 164)

This empiricist view led Schelling fairly naturally to embrace laboratory testing as a useful tool for the economist. In the 1950s experimental methodologies were developed mainly by scholars of different sciences (psychologists, sociologists, mathematicians, philosophers, decision theorists and business-school economists) belonging to a deeply interwoven community, financed mainly by military funds. The role of game theory was to facilitate the transformation of the theories to be tested into simple and precise models. The first experiments showed how laboratory tests could disclose unknown properties of economic behavior challenging also some basic assumptions of mainstream economics. This unconventional orientation of experimental work was promptly perceived as fruitful by Schelling. Indeed, he dedicates a whole chapter of his 1960 book to discuss its usefulness for economics. In this chapter Schelling emphasizes the fact that experimental work helps set the rules for players to use when learning to coordinate their choices in playing the game. Evidence in the lab shows that in the absence of pre-existing rules of behavior, subjects create new norms through a process of learning in which each player's developing norm influences the others. A further implication is that each subject adjusts their own values as a consequence of changes taking place to the values of the other.[11] This process may contribute to solving the unpredictability of the game by taking into account all necessary factors for determining a solution, including unconscious behavior. In this way experimental work can be valuable for defining principles that are relevant to rational play yet are not derivable analytically. This approach of bearing psychological variables and actual behavior in mind, with all the multifaceted components therein, was so pronounced that not only did it attract the critical attention of Harsanyi pointed out in the previous section but it also set off the debate on the axiom of symmetry.

4. The debate on the axiom of symmetry

The initial criticism directed at the assumption of symmetry by most reviewers of Nash's model was that it could only be accepted as an ethical criterion. This view was shared by Luce and Raiffa (1957) in their influential book *Games and Decisions*, as well as by Wagner (1957, 1958) and Shubik (1959), and reprised by Bishop (1963) in a later survey on game theoretical models of bargaining. Luce and Raiffa's arguments are worthy of citing and serve as representative of the dominant view. The close scrutiny of Nash's model by these reviewers points out three major criticisms: one, that the available observational and experimental evidence shows that the model has dubious predictive value, two, that its

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[11] Schelling's comment is related to the "robbers cave experiments" on intergroup conflict and cooperation discussed by Sherif (1958). The other experiments mentioned by Schelling are Bavelas (1953), Deutsch (1958), Flood (1958), and Osgood (1959).

implications are also ethically questionable in that its criteria of fairness is quite context dependent, and three, that the assumption of players who are perfectly informed of each other's utility functions is particularly inappropriate in a bargaining game where players' strategies often strive to falsify personal preferences.

Luce and Raiffa turn this same criticism to Harsanyi's proof of the Nash-Zeuthen equivalence: "It is again difficult to interpret such results either as a descriptive model of human behavior or as a piece of (conditionally) normative counseling. We can, however, see merit in the concession principle as a negotiation scheme which two players might agree is *fair in the abstract* and which they would use to resolve any specific conflict. In this sense, the Harsanyi-Zeuthen result seems to help one accept the Nash solution – or vice versa." (Luce and Raiffa 1957, p. 136)

As Luce and Raiffa see it, although its validity seems to depend on the context Harsanyi's proof is judged more appropriate as an *arbitration scheme*. At best they conclude its normative value is worthy of merit whereas its descriptive validity is seriously questioned.[12]

This matter is addressed by Schelling in a paper entitled *For the Abandonment of Symmetry in Game Theory*, which was first published in 1959 and then included as an appendix in his 1960 book. In this paper Schelling's opinion is quite different from that of his contemporaries. Not only does he not ascribe a normative value to the axiom but he criticizes it for two different reasons: first, the axiom sees a cooperative bargaining game degenerate into a tacit non-cooperative game; secondly, symmetry is only one of the many possible clues for determining a rational solution to the game.

As regards the first criticism Schelling argues that a bargaining game cannot be exclusively defined by its normal form. This holds true also for Nash's (1953) model with variable threats because once the payoffs are known the sequence of moves still have to be specified. In any case the negotiation process, according to Schelling, has to include a final period in which players make the last offer. With the symmetry postulate each player should make this offer without knowing the other player's offer. Once time for communication has expired the bargaining game turns into a tacit game. If this tacit game has a predictable and efficient solution, each player will have an optimal strategy, which is to wait for the last period without reaching an agreement and then converge on the solution of the tacit game. It is indeed rational to expect that the other *symmetrical* player will give nothing further than this. But, following along these lines the cooperative feature of the bargaining game is rendered for the most part irrelevant. Schelling's conclusion is that a bargaining model in which, by virtue of the symmetry assumption, neither pre-play communication nor binding agreement play a part cannot be considered a valuable tool. Moreover, he points out that if the solution to the tacit game is not commonly known it is not possible to define a rational agreement at all.

To support this interpretation Schelling uses a metaphor of a race in which all the runners know that they are equally fast. If the race is certain to end in a tie, the runners do not bother to run: "The perfectly move-symmetrical cooperative game seems a little like that foot race. Bargaining in the one case is as unavailing as leg-work in the other; every player knows in advance that all moves and tactics are foredoomed to neutralization by the symmetrical potentialities available to his opponent. The interesting elements that we might inject in the

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[12] "The Nash solution to the bargaining problem suggests a method of 'fair division'. The best way to look at the motivation behind this method is that it is normative. It suggests a way of dividing joint profits that is 'fair' in the sense that the referee or judge in a cartel board 'should' follow such a procedure if called upon to settle a division between two corporations." (Shubik 1959, p. 49)

bargaining game are meaningless if perfect symmetry, and its acceptance as inevitable by both players, is imposed on the game by its definition." (Schelling 1960, p. 276)

Schelling's second critical remark concerns the fact that even if symmetry may permit actual bargainers to converge on a unique solution, it is not a necessary consequence of the rationality postulate as Harsanyi argues. Schelling points out that the axiom is twofold. On one side it imposes that each player does not concede more than he would expect to obtain if he were in the opponent's place. On the other side it implies that the each player's expectation of what he would concede if he were in the opponent's position is his perception of symmetry. Harsanyi seems to be aware of only the first meaning, which Schelling calls the *rational-solution* postulate, and appears to only implicitly assume the second meaning, the *symmetry* postulate. Schelling explains this by the fact that Harsanyi sees the *rational-solution* postulate as a derivation of the *symmetry* postulate. Schelling counter argues that any alternative division may be rational if it is consistent in terms of shared expectations. For example, the unfair division of 80/20 may be justified if both bargainers see a sign on a blackboard suggesting this proportion or if previous players had agreed on it. Although mathematical symmetry is a useful expedient for finding a solution, its rationality depends on the absence of other available alternatives to focus players' attention on. Indeed, the criteria for choosing an option are not related to mathematical appeal but to introspection and empirical observation.

This interpretation implies that Harsanyi's and Nash's models postulate some degree of limitation in players' processing capacities: "The identification of symmetry with rationality rests on the assumption that there are certain intellectual processes that rational players are incapable of, namely, concerting choices on the basis of anything other than mathematical symmetry, and that rational players should know this. It is an empirical question whether rational players can actually do what such a theory denies they can do and should consequently ignore the strategic principles produced by such a theory." (Schelling 1960, pp. 285-286) In light of this, Schelling's criticism assumes that Nash's axiomatic approach is not based on the abstract assumption of players' rationality but on the empirically wrong assumption that mathematical symmetry dominates all the other possible clues as to how tacitly rational players' expectations may converge on a unique solution. The fact is that only observed evidence can prove its superiority over other coordination devices.

Significantly enough, Harsanyi's reply to Schelling is contained in a paper entitled *On the Rationality Postulates Underlying the Theory of Cooperative Games*.[13] His first argument is that their disagreement depends on their being focused on two different games. Schelling's analysis concerns a kind of game in which players can commit themselves in advance to a certain specified demand and stick to it even in the event of disagreement. In this case ultimatums are effective in reaching a solution only if the penalty is so harsh that any breaking of the commitment would be unprofitable. Here the rational strategy for the proponent player is to demand the maximum admissible share since it will not be in the responder's interests to reject it as long as it leaves him better off than if no agreement is reached at all. Harsanyi, as well as Nash, considers a different game in which the payoffs in the event of disagreement are deterministically fixed by the rules of the game. It is this lack of alternative retaliatory strategies that renders the outcome independent of the *rather accidental* features of the communication system between the bargainers. In these games the axiom of symmetry does not imply, as Schelling claims, that the bargaining game degenerates into a

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[13] In the first lines of the paper Harsanyi writes: "The purpose of this paper is to re-state and re-examine the rationality postulates underlying the theory of cooperative games. It is also proposed to discuss T. C. Schelling's recent criticism of some game-theoretical postulates, in particular the symmetry postulate." (Harsanyi 1961, p. 179)

tacit non-cooperative game. If the players have the same possibilities of communication there is no benefit in their waiting for the very last moment before proposing an agreement. This is because the axiom of symmetry prevents rational players from insisting on extreme demands that will likely turn out to be incompatible. In Harsanyi's view, the optimal strategy for selfish players is to search for a compromise that has to be defined independently on ethical principles.

A similar argument is used by Harsanyi to analyze bargaining games in which players have opposing interests. In this instance, however, the solution depends on the relative power positions of the parts that are assumed to be exogenously determined. To illustrate this point, Harsanyi makes reference to Schelling's example of a river being used as a demarcation line between two conflicting armies. The river flowing across an area to be divided can only solve the tacit problem of coordination but leaves the bargaining conflict unsolved. It is up to the stronger army to decide if the tacitly demarked region is large enough to accommodate its soldiers or if the river has to be crossed. In general, although the outcome of bargaining may be at times affected by irrational behavior, it is reasonable to assume that bargaining leads to agreements resembling the real balance of power between the parties.

Harsanyi also replies to Schelling's second criticism. He argues that the unfair 80/20 division can only be justified by additional environmental variables, such as blackboard signs or precedents explicitly introduced by Schelling. He goes on to say that the key difference between their approaches is not due to the symmetry postulate but rather to the *restriction-of-variables* postulate, which rules out other factors as well. The axiom of symmetry imposes that each part's strategy depends on additional variables only if one part believes that the other's strategy would depend on these same variables. If all the relevant independent variables and the psychological laws governing behavior are taken as common among all bargainers, it follows that there is no rational outcome other than the Nash-Zeuthen solution.

Harsanyi's concluding remarks concern the definition of a rational theory of bargaining. The purpose of the theory is not to show what solution is preferable according to certain exogenous criteria but to predict and to explain the outcome of actual bargaining behavior. As Harsanyi puts it, although irrational behavior may sometimes play a role, in "many cases, at least" it has "only marginal significance." (Harsanyi 1961, p. 193) He accepts that the effort to link strategic interaction and bargaining theory by means of the assumption of symmetry has essentially a normative purpose. But the same can be said for most game theoretical models designed to describe how ideally rational players would act. In order for these models to work in an empirical science like economics, it has to be believed that the assumption of rational and perfectly informed players – Nash's *intelligent and rational* bargainers – has some descriptive value and helps to explain some real facts. Thus, game theory provides a benchmark for explaining actual bargaining processes only if players pursue goal-seeking orientation to a high degree and in a particularly consistent manner. However, where rational bargaining theory proves predicatively inadequate it may still serve as a reference model to define a theory of limited rationality, such as the one that proposed in the same years by Herbert Simon. It can also heuristically explain deviations from the normative conception of rationality or, more simply, point out how departures from Nash's symmetrical solution are explained by differences in bargainers' skills.

Harsanyi's very last consideration accentuates even further the methodological contrast with Schelling: "Even if intrinsically irrelevant variables do play some role in actual bargaining behavior (although it seems to me that Schelling greatly overstates their actual importance) this does not mean that such bargaining behavior represents a rational strategy against a rational opponent – at least in cases where rational bargaining strategies not involving such irrelevant variables are available. A concept of rational behavior broad enough

to include bargaining strategies based on irrelevant variables would be, in my view, too vague to be of much use for analytical purposes." (Harsanyi 1961, p. 194)

This quotation makes clear that the definition of rationality is a crucial problem raised by the controversy on the symmetry axiom. Thus, the historical meaning of the episode can be better appreciated by focusing on the underlying methodological issues.

A first theme is related to the implications of symmetry for the conception of the individual. If different bargainers are formally treated as if they were not distinguishable one from the other, the model abstracts from real individuals and typifies them into artificial beings. Such an assumption is in line with the neoclassical concept of representative agent which does not necessarily commit to the assumption that all agents in the population are identical, due to the fact that they are univocally defined and differentiated in terms of their preferences. Game theory adopts this same abstract view of the individual by determining solutions without defining the identity of the players who are described solely in terms of strategies available to them. Nash's theory too explicitly endorsed this view. Since the 1950s Harsanyi was more sympathetic with a different interpretation, which anticipates what Aumann (1976) later identifies as the Harsanyi doctrine. The doctrine states that the only way for a player to predict other players' strategies is to rely on his own thoughts and calculations. When individual beliefs in different states of the world are the posteriors they form from a common prior given their private information, introspection becomes the best tool to predict other players' beliefs. This approach however does not resolve the problem of how the common prior is formed,[14] although it permits to erect game theory on the assumption of common knowledge.

Schelling on the other hand deals with strategically interactive environments in quite a different way. His theory of conflict is based on the hypothesis that bargainers may have different abilities and divergent expectations and consequently adopt different strategies. Coordination cannot just be a matter of guessing what an average man does because only heterogeneity among players permits to fully take into account strategic interaction. The task of forming and coordinating players' expectations cannot be left solely to introspection nor to externally defined social conventions but can only be left to the endogenously originated framework where the bargaining game is strategically played. The consequence of this view is that there is no deterministic way to define either the equilibrium solution or the rational behavior.

Indeed, the multiplicity of solutions is another key issue raised by the debate. Harsanyi's most recurrent argument in support of Nash's model is that it determines a single solution to the bargaining game. Nash describes the process conducing him to select this solution in the following way: "Rather than solve the two person cooperative game by analyzing the bargaining process, one can attack the problem axiomatically by stating general properties that *any reasonable solution* should possess. By specifying enough such properties one excludes all but one solution." (Nash 1950, p. 136)

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[14] In the '60s, Harsanyi seemed to believe that the exercise of imagining opponents' reasoning in a bargaining process may also be based on a common social and cultural framework. This view was made explicit in the following passage: "On the one hand it is conceivable that in a given society with well-established cultural traditions people tend to *enter* bargaining situations with more or less consistent expectations about each other's utility functions. It may happen that *all* members of a given society are expected to have essentially the *same* utility function. Or, more realistically, we may assume that at least persons of a *given* sex, age, social position, education, etc. are expected to have similar functions of a specified sort." (Harsanyi 1962, p. 33) This view also assumed a cold-war flavor: "In the case of bargaining between Communist and non-Communist powers all of these difficulties seem to appear in rather extreme form." (Harsanyi 1962, p. 38)

Harsanyi emphasizes the importance of the method by reversing its sequence: if the *relevant* independent variable alone allows for the defining of a unique rational solution, there is no need to consider other variables except those "intrinsically irrelevant for utility maximization." (Harsanyi 1961, p. 190) This emphasis is a consequence of the focus on the efficiency properties of the final outcome. In Nash's model bargainers communicate only by addressing demands to each other without any explicit process of commitment and players' bargaining power are axiomatically determined.

On the other hand, Schelling's theory of conflict considers players' bargaining power as an endogenous variable dependent on what happens in rather than out of the negotiation process. In games with multiple equilibria, it is assumed that players learn what is convenient to do by relying on directly relevant past experiences or clues given by the actual bargaining process. For instance, in Schelling's discussion bargainers do not try to assess the opponent's estimate of the risk of a conflict as in Zeuthen (1930), but try to affect it by means of all available means. Bargaining becomes a subtle communication game where messages and information conveyed during the whole negotiation process are instrumental in determining the final outcome. It is also clear that in such a game the moves that each part has made to ascertain the preferences of the counter part become relevant. This problem, which Nash and Harsanyi avoid by assuming complete information, is explicitly considered by Schelling. For example, each bargainer may try to conceal his own preferences. Or, he may reveal false preferences to manipulate the others. Or still he may acquire information by observing the others' actions. As Schelling sees it, the attempts to manipulate the bargaining environment are often aimed at rendering other players' choices more predictable by influencing them. This corresponds to gaining as much influence or leverage over other players' behavior as possible also by controlling the information used by them. It is evident how this process directly affects players' relative power and makes the solution endogenously determined.

Schelling's strongest criticism of Harsanyi however is that there isn't a unique and mechanic way to define rationality. The only way to define an unambiguous rational criterion of choice for strategic environments is to rely on the process through which players learn to predict other players' choices. While Harsanyi assumes that all rational players make the same maximizing choices under the same circumstances, Schelling points out that the use of the maximization principle may not always be rational. Rational bargainers consequently are not constrained to adopt only one pattern of behavior: "Rationality is a collection of attributes, and departures from complete rationality may be in many different directions. Irrationality can imply a disorderly and inconsistent value system, faulty calculation, an inability to receive messages or to communicate efficiently; it can imply random or haphazard influences in the reaching of decisions or the transmission of them, or in the receipt or conveyance of information; and it sometimes merely reflects the collective nature of a decision among individuals who do not have identical value systems and whose organizational arrangements and communication systems do not cause them to act like a single entity. As a matter of fact, many of the critical elements that go into a model of rational behavior can be identified with particular types of rationality or irrationality." (Schelling 1960, p. 16) It is quite clear that this view was in stark contrast with the 1970s shift to the non-cooperative approach that made game theoretical application to economics strictly dependent on Nash equilibrium. Nash assumes that players are endowed with perfect knowledge and unlimited computational power while bypassing the problem of determining why they should come to play their equilibrium strategies (Giocoli 2004). In this setting Schelling's theory is simply unfit to be accepted within mainstream economics.

To summarize the methodological content of the Harsanyi-Schelling, the assumption of symmetry plays a key role in the foundation and in the subsequent developments of the

rational theory of strategic bargaining. First, the axiom enables one to sidestep the main problem of strategic interaction, that is to define the process through which players define their expectations on opponent's behavior. The theory considers only the end state of the equilibrium and ignores the negotiation process. Second, it extends the abstract assumption of 'representative agent', that is a constitutive notion of neoclassical economics, to bargaining theory. Third, it enables one to solve the problems associated with the multiplicity of equilibria by selecting a unique solution to the bargaining problem.

The above considerations illustrate what strategic bargaining theory has had to face since its inception more directly than other research fields: the problem of finding a stop to the infinite regress that typically characterizes situations where the outcome for each part depends upon the expectations of the other parts. Before game theory, this problem was solved parametrically. Parts were assumed to assess opponent's beliefs probabilistically as postulated by Zeuthen's model. Nash and Harsanyi however attacked the same problem by introducing the axiom of symmetry which ignored the process through which those probabilistic expectations are actually determined. But the cost is high for this way out. First, bargaining theory must be taken back within the realm of static analysis by excluding any dynamic adjustment or correction of these expectations over time. Second, empirical significance has to be subtracted from the model. The alternative path proposed by Schelling was to avoid introducing such requiring assumptions and tried to preserve strategic interaction in its complexity. The consequence was that he gave up the search for determinate solutions and assigned all the work to be done to empirical generalizations.

5. Conclusions

This paper has argued that the Harsanyi-Schelling debate on the assumption of symmetry was a historically significant episode for the evolution of bargaining theory. Contrary to most reviewers of Nash's bargaining model, Schelling did not consider the axiom suitable only as an ethical criterion. Schelling criticized it without rejecting Harsanyi's rational approach but showing its inadequacy by means of counterexamples and paradoxes. His approach of taking into account the empirical content of theory pushed him to consider the framework in which the bargaining game is played as the direct object of analysis. Normative precepts therefore were eventually built upon the features of the actual environment in which they apply. The assumption of symmetry was consequently seen by Schelling as a tool to exclude the negotiation process from the bargaining model. In this way the key issue of the learning process through which each player defines his expectations of the behavior of the counterpart was ignored. In place of this, Harsanyi extended the neoclassical assumption of the representative agent to bargaining theory and removed any heterogeneity among players. These assumptions lead to a unique outcome to the bargaining game.

This debate also helps to explain why Nash's bargaining model has come to exert a great influence on recent work and why the axiomatic approach prevailed over other ways of strategically modeling the bargaining process. On one hand, Harsanyi is the economist who permitted the application of game theory to situations in which the players lack complete information about each other or the rules of the game. The common prior assumption that he proposed to deal with this issue is probably the most pervasive assumption in the economics of information and is strictly related to Harsanyi's interpretation of the axiom of symmetry. Moreover it allowed for the introduction in bargaining theory of the same assumption of strong rationality which characterizes the non-cooperative solution concept, namely Nash equilibrium. All bargainers are assumed to be perfect maximizers on the basis that the

opponents do the same. In this way their strategic behavior becomes easily predictable and solutions can be defined axiomatically by evading the issue of the empirical plausibility of their theories. The only empirical findings mentioned by Harsanyi in support of the symmetry assumption are preliminary to the model and concern the supposed prevalence of compromises among rational bargainers. Once the validity of this rule is accepted, the axiomatic model does not allow for contradictions apart from internal consistency. On the other hand Schelling's work on bargaining theory has not been fully developed mainly because Schelling adopted a multifaceted definition of rationality. If players do not share exactly the same pattern of behavior, outcomes become indeterminate in an abstract framing of analysis. The process of finding an agreement in a bargaining process, similarly to that of selecting the equilibrium in a non-cooperative game, has a component that is inherently empirical because it concerns a process of intellectual coordination among heterogeneous agent where the context is decisive. This approach greatly contrasted with the post-1970s emphasis on perfectly informed and fully rational economic agents and on equilibrium analysis.

In conclusion, these arguments show that bargaining, more than other research fields, is a problematic issue for economists. It highlights well the anomalies contained in the generally agreed definition of rationality and in the surreptitious division between formalized and empirical arguments characterizing mainstream economics. More generally, this historical evidence also questions that to ground a new research field on highly stylized assumptions and on the early achievement of determinate solutions as primary goal is necessarily the best strategy in order to improve the future efficacy of a research programme.

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