Tax Evasion and Coordination

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Abstract

We consider corporate tax evasion as a decision affecting business partners. There are costs of uncoordinated tax reports, both in terms of catching inspectors' attention and running accounts. If these costs are small, there exist a unique Nash equilibrium of the game between the tax authority and a population of heterogenous firms. In this equilibrium, the miscoordination costs enhance non-compliance if and only if more than 50% of the firms are cheating. This provides one rationale for developing countries to be cautious with employing refined auditing schemes and for developed countries to promote complicated accounting procedures.

JEL Classification: H26, H32

Keywords: tax evasion, coordination, business partners

1 Introduction

Recent years have seen a surge in research on tax evasion of firms. The interest was aroused by an observation that firm adds new dimensions to the problem over and above standard gambling and cat-and-mouse[1] approaches. First, a firm is not a single decision maker and has its own agency problem, as stressed by Crocker and Slemrod (2005). Second, the interaction between firms can be important for the

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[1] The term is borrowed from Cowell (2006) and refers to the modeling of evasion as a game between tax agency and a single taxpayer.

general outcome, as Bayer and Cowell (2005) and Sanchez (2006) point out, although Lipatov (2003) shows that the interaction matters in games with individual taxpayers as well.

We look here at a long run situation in an economy where firms exercise transactions with each other. Each firm submits a tax report that can be then audited by the tax authority. The firms differ in their decision to evade taxes and if so, how aggressively. This is a long run decision, as it requires adoption of special accounting policy that either permits evasion or not. By evasion we certainly mean sophisticated evasion, that is tax evasion that requires certain expertise and involves intricate manipulation of accounts, as opposed to blunt underreporting. This term is also used in the same sense by Lipatov (2005). The evasion decision of the firms is an outcome of rational profit maximization given their expectations about the decision of transacting partners and audit intensity.

Thus, in our economy the firms face two types of costs in addition to standard costs and benefits of evasion. We call them coordination costs. The first type is exogenous costs, which arise every time there is a transaction between firms with different decisions about evasion. These are related to the adjustment of accounts for different kinds of firms: e. g., an evading firm that meets an "honest" firm has to create an additional fictitious firm in order to evade without creating inconsistency in the accounts, whereas two evading firms can manage the transaction consistent without additional efforts. The second type is endogenous costs, which arise every time the tax authority sets unequal probability of auditing for the cases of observing similar and different reports of the two transacting firms.

The endogenous costs are also present in Sanchez (2006). The difference of his paper from our approach is not only in lack of exogenous costs, but also that he considers tax authority with ability to commit. This is well explained by different ideas underlying the two papers: whereas we consider long-run equilibrium, Sanchez concentrates on the short-term with the aim of constructing auditing rule that minimizes mistakes of the tax authority (in sense of auditing the honest and not auditing cheaters). Furthermore, whereas Sanchez describes the situation in a homogenous auditing class, assuming perfect correlation of income and uncertainty about the auditing rule, we consider a pair of firms with imperfectly correlated income.

The paper by Bayer and Cowell (2005) stands even further from us, as it looks at the effect of auditing on joint decision of competing firms to evade and to produce.

We consider firms that are partners rather than competitors. Crocker and Slemrod (2005) go inside a firm, whereas we treat it as a decision making unit.

In general, we believe that no commitment approach is more appropriate for the models with two levels of income. Firstly, the tradition in the literature is that commitment models are only considered with a continuum of income levels. Secondly, though the auditing rules are often announced, there is no means to establish whether they are actually followed. Thus, ability to commit may be a too strong assumption to make.

The tax authority in our model observes the transacting pairs. This seems a reasonable assumption at least for Russia, where the auditing of one firm involves checking accounts of the firms that are transacting with it, as described e. g. in Sumina (2006). In reality there are many firms transacting with each other, and taking pairs is just a necessary simplification. Considering more than two firms in a match and overlapping matches would make the analysis unmanageable analytically, while not adding much to our main point.

The main result of the paper is the fact that the equilibrium cheating and auditing differ substantially from the approach disregarding transactions among the firms, even if the costs of miscoordination are small. When evasion is not popular (less than a half of the firms evade), the share of cheating firms as well as the auditing probability are likely to be overestimated, if the coordination of tax reports is not taken into account. In case of popular misreporting, both the share of non-compliers and the auditing probability are underestimated. It is worth noting that the auditing probability in our setting varies with the reports combination, making comparison with uniform auditing probability of the representative case difficult in principle.

Furthermore, we find that the miscoordination costs decrease cheating and auditing when less than 50% of all firms are underreporting and increase them in case evasion is popular. The correlation of profits has a similar effect. In both instances, with coordination cost ascent the more popular strategy becomes more attractive, hence more firms choose it in equilibrium.

The auditing probability in our model can be positively affected by the amount of fines, unlike in representative case. This becomes possible because the direct effect of larger fines to make auditing more attractive overplays the indirect effect coming through the reduced cheating. Coordination costs amplify the indirect effect, making fines more or less effective depending on whether cheating or honesty prevails. We also show that the auditing is negatively affected by its own costs. All three types of costs, exogenous miscoordination, profits correlation embodied in different auditing probabilities, and auditing costs are reinforcing each other.

We also shed some light on the mechanism of evasion game when coordination matters: we show that correlation of profits solely generates the difference in auditing probabilities. The exogenous miscoordination costs alone change equilibrium cheating and auditing, but leave the latter independent from the report configuration.

The rest of the paper is structured as follows. The model setup is presented in the next section, followed by the description of equilibrium structure. Section four is devoted to the discussion of the results for mixed equilibrium. Section five looks at a calibrated example. Conclusion is followed by appendix with derivations of equilibria and results.

2 Evasion game

2.1 Single firm benchmark

Let us start with the case when there are no transacting pairs and no miscoordination costs. A single firm decides whether to evade its profit, facing the tax authority that can perform auditing. We use the approach of Graez, Reinganum and Wilde (1986) in this benchmark, with a convex rather than linear cost function for auditing.

First, the nature moves, assigning a type to the firms: high profit $h = \pi$ or low profit $l = 0$. The types are drawn from a distribution characterized by a density function

$$f(x) = \begin{cases} \gamma & \text{if } x = \pi \\ 1 - \gamma & \text{if } x = 0 \end{cases} .$$

Second, the high profit firms decide whether to submit a high report $H = \pi$ (be honest) or a low report $L = 0$ (cheat).

The tax authority does not audit high reports and exerts effort a in auditing low reports. We take a function $a(p) = -k \ln(1 - p)$ from Reinganum and Wilde (1986) as the mapping from detection probability defined on the unit interval to the auditing effort defined for non-negative real numbers. The inverse function determines detection probability from the effort $p(a) = 1 - e^{-\frac{a}{k}}$. k is a detection difficulty parameter: the higher it is, the more effort is required to support a given detection probability. The firms can never be detected with certainty, and zero effort results in zero detection probability. The low report is honest with probability $\frac{1-\gamma}{1-\gamma+q\gamma}$ and not with the complementary probability, where q is the probability that high profit firm is cheating.

The authority is maximizing its expected revenue $\frac{q\gamma}{1-\gamma+q\gamma}p\left(a\right)\left(1+s\right)t\pi-a$, the high income firm - its expected profit $\pi-p\left(a\right)\left(1+s\right)t\pi$. Here s is a surcharge rate for being caught, t is a tax rate.

When $st\pi>\frac{k}{\gamma}$, there exists a unique equilibrium characterized by the auditing effort a^* and the evasion probability q^*:

$$a^*=-k\ln\frac{k\left(1-\gamma+q^*\gamma\right)}{q^*\gamma\left(1+s\right)t\pi}, \tag{1}$$

$$q^*=\frac{k}{st\pi-k}\frac{1-\gamma}{\gamma}. \tag{2}$$

This is a conventional result: the evasion share is increasing in auditing costs and decreasing in the share of high income taxpayers and the fine bill.

When $st\pi\leq\frac{k}{\gamma}$, a unique equilibrium is characterized by $q^{fc}=1$ and $a^{fc}=-k\ln\frac{k}{\gamma(1+s)t\pi}$.

Thus, the equilibrium is always unique. It is a mixed equilibrium, when the fine is large relative to the auditing costs per high income taxpayer. If, to the opposite, the fine bill is smaller than the auditing cost, the firm plays pure strategy of cheating in equilibrium. In our benchmark firms never submit high report with probability one. The mixed equilibrium is of most interest to us, since the fines are usually high enough to cover auditing costs in reality. Moreover, this mixed equilibrium is evolutionary stable (Weibull 1995), as even if a small part of taxpayers gives honest reports, the reduction in detection probability is not enough to off-set a loss from lower evasion.

2.2 Two transacting firms

For further analysis it is useful to introduce the following terminology:

Definition 1 *We call an equilibrium of our game* full cheating*, if all the firms are submitting low (zero) reports in this equilibrium* $q^*=1$*; we call an equilibrium* full honesty*, if all the high income firms submit high reports* $q^*=0$*.*

2.2.1 General setup

Consider a simultaneous game between two risk neutral firms (call them, for example, a buyer and a seller) and a tax authority.

The first move is made by the nature that assigns a type to each of the two firms: high profit $h = \pi$ or low profit $l = 0$. We assume now that the profits are correlated with the correlation coefficient $r, 0 \leq r < 1$[2]. We do not consider negative correlation, as our firms are cooperating rather than competing. The joint distribution of two types in a match is given by the following density function:

$$f(x, y) = \begin{cases} \delta, & \text{if } x = y = \pi, \\ \gamma - \delta, & \text{if } \{x, y\} = \{0, \pi\}, \\ 1 - 2\gamma + \delta, & \text{if } x = y = 0. \end{cases}$$

where $\delta := \gamma^2 + \gamma(1 - \gamma) r$.

The second move is made by the high profit firms. They decide whether to submit a high report $H = \pi$ (be honest) or a low report $L = 0$ (cheat). A high income firm incurs exogenous coordination costs c if the other firm is of the same type but submits a different tax report. Each firm of type h (high profit) gets expected payoff of $u(i, j)$, where i is its own report and j is a report of its partner:

$$\begin{aligned} u(L, L) &= \pi - p(LL)(1 + s) t\pi, \\ u(L, H) &= \pi - p(HL)(1 + s) t\pi - c, \\ u(H, H) &= u(H, L) = \pi(1 - t). \end{aligned}$$

The firm of type l gets zero payoff.

The third move is by the tax authority, which chooses an auditing effort $a \in R_+$ conditional on the reports observed: $a(LL)$ (two low reports), $a(HL)$ (a low and a high report in any order), $a(HH)$ (two high reports). The tax authority gets expected revenue of $p(a)(1 + s) t\pi - a$ from each cheater it audits and the revenue $t\pi - a$ from each honest report it audits.

The game takes into account both exogenous costs (c per firm) and endogenous costs of miscoordination. The exogenous costs have at least two sources: 1) it takes up resources to create an evasion scheme, so sharing the evasion design costs is one way to reduce per firm costs; 2) it is easier to run accounts of each firm (they are more compatible), when both and not only one are evading. If the first source plays an important role, the exogenous coordination costs are greater for the firm which is

[2] We have also analyzed the case when $r = 1$, but since this is not likely to happen in reality, we do not present the results here. It turns out that the equilibrium structure in this case is distinctly different from correlation arbitrary close to perfect, so we also can not use it as a benchmark. The derivation of equilibrium is available apon request.

evading, as the honest one does not have to develop an evasion scheme. For the sake of simplicity though we let the two equal in the present analysis, as it does not alter the main message of the paper, coordination aspect of firms' evasion. One can think that the evasion schemes are easily available in the economy, so that the first factor becomes unimportant.

The endogenous coordination cost is self-explanatory: it reflects the difference in detection probabilities the tax authority might want to generate. Namely, it can exert different efforts in auditing low profit report depending on whether it comes with another low report or with a high report. Compared to the case of two low reports, it needs a half of resources to provide the same auditing probability if one of the reports is high. Thus, we do not consider that coordinated evasion might require more effort to discover than uncoordinated.

We choose the simultaneous formulation rather than a sequential one, because we do not want to consider a particular industry structure or a relation between an entrant and an incumbent. Our goal is to characterize the economy where two firms from different populations (again, think of buyers and sellers) meet to play a coordination game. Even more, since the decisions are long-term, they become a property of the firms, so that they can be characterized as evaders or honest. In this way, the Nash equilibria of the simultaneous game show us where these populations could converge, if, for example, less profitable firms were dying out.

2.2.2 Optimization problem of the tax authority

The tax authority observes the match. Recall that we denote with lower-case letters the profits, and with upper-case the reports. We have then the following profit - report table

	total	HH	HL	LL
hh	δ	$\delta(1-q)^2$	$2\delta q(1-q)$	δq^2
hl	$2(\gamma-\delta)$	0	$2(\gamma-\delta)(1-q)$	$2q(\gamma-\delta)$
ll	$1-2\gamma+\delta$	0	0	$1-2\gamma+\delta$

which represents the measures (or shares) of taxpayer pairs reporting incomes given by the column entries, while actually receiving incomes given by row entries.

The following lemma characterizes the best response of the tax authority in this case.

Lemma 2 *In the tax evasion game above the best response of the tax authority to the firms cheating with probability $q \in (0,1]$ is the following strategy:*

$$a(HH) = 0, \tag{3}$$

$$a(HL, q) = -k \ln\left(k \frac{\delta q + \gamma - \delta}{\delta q (1+s) t\pi}\right), \tag{4}$$

$$a(LL, q) = -k \ln\left(k \frac{\delta q^2 + 2q(\gamma - \delta) + 1 - 2\gamma + \delta}{(\delta q^2 + q(\gamma - \delta))(1+s) t\pi}\right). \tag{5}$$

The proof is left to the appendix A. Obviously, observing two high reports the tax authority does not audit them. Observing different reports in a match, the authority audits the low one with probability determined by the effort $a(HL)$. When two low reports are observed, the optimal auditing effort is given by $a(HL)$.

Note that the two efforts (and corresponding probabilities) are only equal, when $r = 0$, that is the report of one firm does not contain any information about the profit of the other firm. With $r > 0$ we have $a(HL) > a(LL)$, which is quite intuitive: different reports indicate possible cheating, so it makes sense to audit them more.

2.2.3 Equilibria

The proposition 3 characterizes the equilibria arising in case of perfectly correlated draws. We denote the equilibrium values of cheating probability with q^* and of auditing effort with a^*.

Proposition 3 *In the tax evasion game with two transacting firms*

(i) If $\delta c < \min\{\gamma s t\pi - k, \gamma t\pi\}$, there exists a unique symmetric equilibrium, and

$$q^* = \frac{\gamma s t\pi + \delta c - k\gamma - \sqrt{(\gamma s t\pi + \delta c - k\gamma)^2 - 4\delta c k (1-\gamma)}}{2\delta c}, \tag{6}$$

$a^*(HH) = 0$, $a^*(HL) = a(HL, q^*)$, $a^*(LL) = a(LL, q^*)$ as given by (3).

(ii) If $\gamma s t\pi - k \leq \delta c < \gamma t\pi$, there exists a unique full cheating equilibrium, and $q^* = 1$, $a^*(HH) = 0$, $a^*(HL) = a(HL, 1)$, $a^*(LL) = a(LL, 1)$.

(iii) If $\gamma t\pi \leq \delta c < \gamma s t\pi - k$, there is a unique equilibrium of full honesty, and $q^* = 0, a^* \equiv 0$.

(iv) If $\delta c > \max\{\gamma s t\pi - k, \gamma t\pi\}$, there are three equilibria: full honesty described in (iii), full cheating described in (ii), and a mixed equilibrium described in (i).

The proof of the proposition is left to appendix B. The structure of equilibria is very intuitive: for small coordination costs (relative to both payoff from auditing cheaters $\gamma st\pi - k$ and the tax bill $\gamma t\pi$) there is a unique mixed equilibrium, as in a standard game without coordination issues. With larger costs of miscoordination, pure cheating or pure honesty dominates depending on how large are auditing costs relative to auditing benefits (fines and share of high income firms). Finally, when the miscoordination costs are very large, any coordinated pure strategy profile is an equilibrium, plus there is an unstable mixture between them.

3 Discussion of the results

3.1 Summary

Since we believe that the exogenous coordination costs are relatively small, we can concentrate on the regions of parameter values where a mixed equilibrium exists. Then for further consideration our results can be conveniently summarized in the following table:

	representative	correlated
q^*	$\frac{k}{st\pi-k}\frac{1-\gamma}{\gamma}$	$\frac{\gamma st\pi+\delta c-k\gamma-\sqrt{(\gamma st\pi+\delta c-k\gamma)^2-4\delta ck(1-\gamma)}}{2\delta c}$
$p^*\,(LL)$	$\frac{1}{1+s}$	$1-k\frac{\delta q^{*2}+2q^*(\gamma-\delta)+1-2\gamma+\delta}{(\delta q^{*2}+q^*(\gamma-\delta))(1+s)t\pi}$
$p^*\,(HL)$	n/a	$1-k\frac{\delta q^*+\gamma-\delta}{\delta q^*(1+s)t\pi}$

Here we used (1) to derive the auditing probability in representative case. The other expressions are taken straightforwardly from the text. As it has been already noted, the probability of auditing for dissonant reports is higher than that for the same reports as long as $r > 0$. A further breakdown of the miscoordination costs propagation mechanism is represented in the table below:

	$c=0, r=0$	$c>0, r=0$	$c=0, r>0$
q^*	$\frac{k}{st\pi-k}\frac{1-\gamma}{\gamma}$	$\frac{st\pi+\gamma c-k-\sqrt{(st\pi+\gamma c-k)^2-8ck\gamma(1-\gamma)}}{4\gamma c}$	$\frac{k}{st\pi-k}\frac{1-\gamma}{\gamma}$
$p^*\,(LL)$	$\frac{1}{1+s}$	$1-\frac{k(1-\gamma+q^*\gamma)}{q^*\gamma(1+s)t\pi}$	$1-k\frac{1-2(1-q^*)\gamma+\delta(1-q^*)^2}{(\delta(q^*-1)+\gamma)q^*(1+s)t\pi}$
$p^*\,(HL)$	$\frac{1}{1+s}$	$1-\frac{k(1-\gamma+q^*\gamma)}{q^*\gamma(1+s)t\pi}$	$\frac{1}{1+s}$

From this table we see clearly that the endogenous miscoordination costs are only embodied in the differential auditing probability, and some correlation in the profits is enough to generate them even in the absence of exogenous costs. On the other hand, only exogenous costs shift equilibrium cheating probability even in the absence of auditing intensity differential. Thus, the two channels of the miscoordination costs can be clearly separated. This does not mean they do not interact at all; in fact, comparative statics shows that they reinforce each other.

3.1.1 Payoffs

With each case there are associated payoffs: a pair of tax revenues R and after-tax (expected) income of high income firm I. We summarize them in another table

	representative	correlated
R	$\gamma t\pi - (1-\gamma+q^*\gamma)\,a$	R_c
I	$(1-t)\,\pi$	$(1-t)\,\pi - \frac{\delta}{\gamma}q^*c$

where

$$\begin{aligned} R_c/2 &= \gamma(1-q^*)\,t\pi + q^*\gamma(1+s)\,t\pi - k\,(q^*\gamma+1-\gamma) - \\ &\quad (\delta q^*+\gamma-\delta)(1-q)\,a\,(HL) - a\,(LL)\left(\delta q^{*2} + 2q^*(\gamma-\delta) + 1 - 2\gamma + \delta\right). \end{aligned}$$

The derivation of each term is left to the appendix C.

We see that with more correlation the expected punishment for miscoordination is more severe. This is due to the absence of coordination costs in case of matching with a low profit firm. The revenue of the tax authority is influenced through many different channels, e.g evasion share and two auditing probabilities, so it is impossible to say something unambiguous at this level of generality.

3.2 Comparative statics

Our next step is to derive comparative statics results in the same vein. First, we are interested how effective the fines are in deterring firms from cheating, second, how the equilibrium values depend on the coordination costs. The derivation is left to the appendix D.

	representative	matched
$\frac{dq^*}{dc}, \frac{dq^*}{dr}$	n/a	$>< 0$
$\frac{dq^*}{dk}$	> 0	> 0
$\frac{dq^*}{ds}$	$-\frac{kt\pi}{(st\pi-k)^2}\frac{1-\gamma}{\gamma} < 0$	< 0
$\frac{dp^*}{ds}$	$-\frac{1}{(1+s)^2} < 0$	< 0

We can see that miscoordination costs affect equilibrium compliance in an interesting way. If the share of compliant taxpayers is above one half, the costs are decreasing evasion; in the opposite case they are increasing it. This is true for both exogenous and endogenous costs. The second order effect is also crucially affected by popularity of cheating: the cheating is convex in costs for intervals $\left(0, \frac{1}{4}\right)$ and $\left(\frac{1}{2}, 1\right)$ of the range; it is concave on the rest of the unit interval. In other words, if the cheating is popular, it becomes increasingly so with higher miscoordination costs. The auditing costs make the cheating easier in any case.

The intuition here is straightforward: when cheating is more popular, a firm can economize on the miscoordination costs by "imitating" behavior of the majority. The auditing costs do not depend on the companion-cheaters, so their effect on compliance is constant negative. The convexity results are not so straightforward, and reflect the nonlinearity of profit maximizing relation between miscoordination costs and compliance.

We also look at the interaction of all three types of costs. The general tendency here is reinforcing each other, that is the cheating stimulating effect of, for instance, miscoordination costs is the greater the larger are auditing costs. We shall get the following kind of matrix for our interactions,

$$\begin{array}{cccc} & c & \delta & k \\ c & + & & \\ \delta & + & + & \\ k & + & + & + \end{array}$$

where 'plus' means reinforcing effect.

The other block of comparative statics results is related to fine. The cheating is of course decreasing in fine. We define the measure of effectiveness of the fine as the absolute value of the derivative of the equilibrium cheating $\left|\frac{dq^*}{ds}\right|$. We are interested how this measure is affected by costs. Unexpectedly, the fine effectiveness is increasing with miscoordination costs when cheating is popular, and decreasing in the opposite case. This fact becomes more intuitive when we see that the fine effectiveness is increasing in cheating itself. Thus, the costs affect fine effectiveness through the amount of cheating.

Immediate policy advice follows. If we observe a change in the level of miscoordination costs (for example, as a result of Sorbane-Oxley act they rise dramatically) given popularity of cheating, a change in fine effectiveness can be predicted, and hence we can say whether the fines should be corrected. Another little matrix illustrates this logic:

$$\begin{array}{ccc} & q^* \simeq 1 & q^* \simeq 0 \\ c\uparrow & + & - \\ c\downarrow & - & + \end{array}$$

The last effect we consider here is the one of the share of high income taxpayers. Though more subtle and complicated for an arbitrary value of cheating, this effect is the same as the cost effect at extremes: when the cheating is very popular, more high income taxpayers increase the share of cheating; when it is very rare, the opposite is true. The mechanism works also through coordination: with a larger share of high income taxpayers there are more miscoordination costs to economize on, that is "imitate" the majority.

3.3 Example

In this example we calibrate our parameters to the values common in the literature. We want to see how at plausible parameter values the coordination costs affect equilibrium cheating and auditing quantitatively. To do this, we shall firstly explain the choice of parameters. Secondly, we define two benchmarks according to how widespread evasion is: popular cheating featuring developing countries and rare cheating characterizing developed world. Finally, we look at how the cheating and auditing probabilities as well as tax revenue are changing for each of the benchmarks.

3.3.1 Cheating probability

Since the literature before us did not consider miscoordination costs explicitly, we leave them free. We take the values of most parameters directly from Lipatov (2005), as we follows the same logic there: $s = 0.8$, $t = 0.3$, $\gamma = 0.5$. Choice of π is arbitrary, as it is not unit-free. We normalize the profit to unity to have $\pi = 1$. The simplest calibration for the case of no miscoordination costs gives

$$k = \frac{1}{\frac{1-\gamma}{q^0\gamma} + 1} st,$$

which having in mind estimates for shadow sector of 60% $q^0 = 0.6$ in some countries gives $k = 0.045$. Fixing these parameters, we get the following picture:

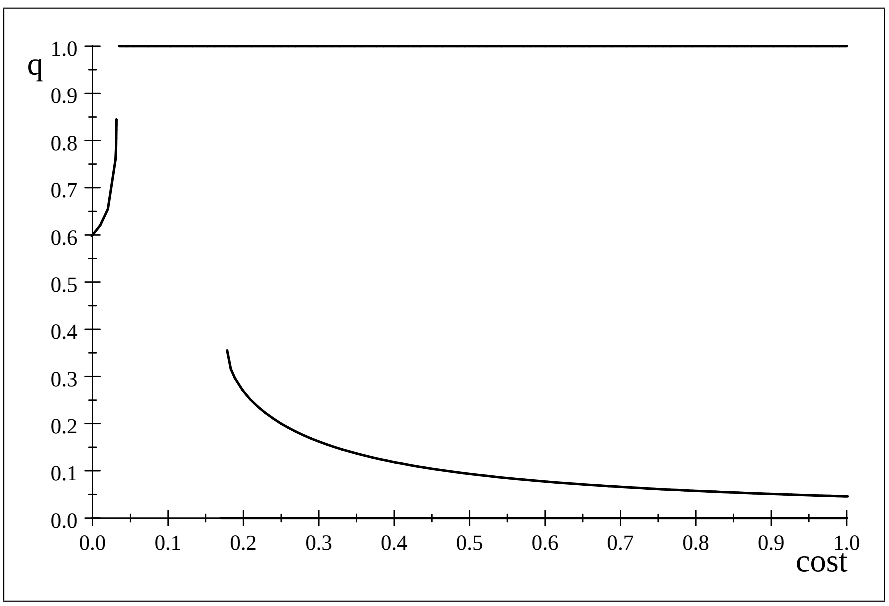

Figure 1. The share of cheating firms q depending on miscoordination costs δc, high evasion regime

On the horizontal axis we can see here 'joint' miscoordination costs, that is a product of exogenous costs c and profit correlation r. The vertical axis show the share of cheating firms. Note that for given value of high income share, δ takes values between 0.25 and 0.5, so that most of variation observed on the picture is due to the exogenous costs.

The chart illustrates all types of equilibria considered before: with low miscoordination costs there is a mixture, in which the costs are increasing cheating; with higher costs there is full evasion equilibrium, and at very high costs three equilibria exist.

We can see that if the auditing costs are low enough so that without miscoordination the cheating is less than a half (we take 20% $q^0 = 0.2$), we get a different picture:

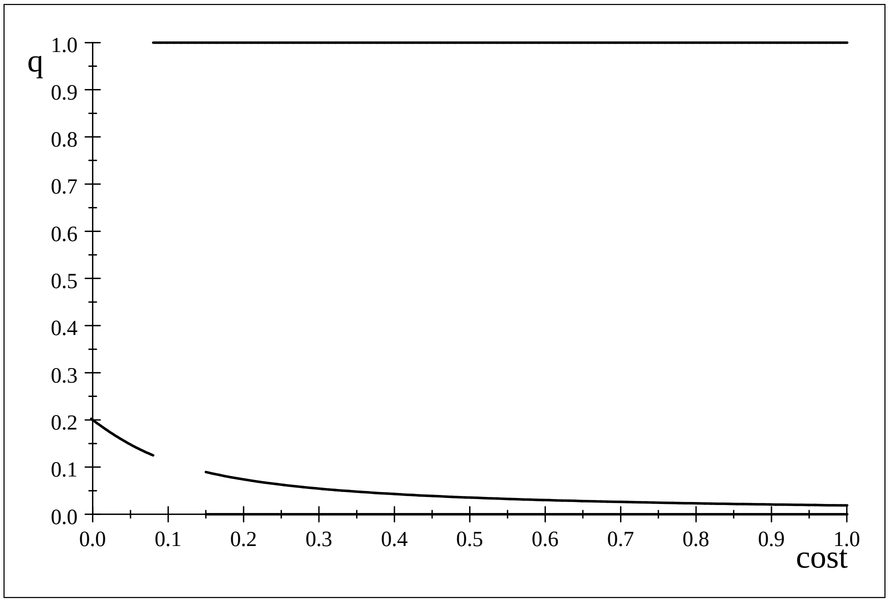

Figure 2. The share of cheaters q depending on miscoordination costs δc, low evasion regime

Thus, in our calibrated example with low auditing costs the mixed equilibrium is very robust to the miscoordination costs changes. We should note, however, that the values of cheating share for high miscoordination costs are not precise, as the lower auditing probability hits nonnegativity constraint (below the range is shown more precisely). As predicted, the costs decrease cheating when it is not popular.

3.3.2 Auditing probability - high low reports combination

The auditing probability for the both cases is plotted on the following pictures:

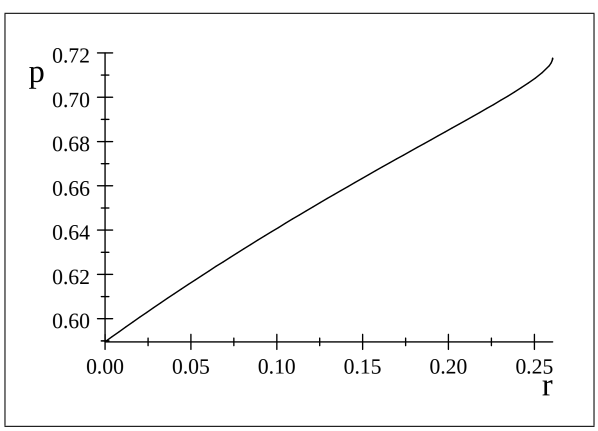

Figure 3. High low reports auditing probability $p(HL)$ depending on correlation r, high evasion regime, $c = 0.1$

This is the higher probability (to audit a miscoordinated report) for a situation when cheating is popular. We plot the correlation coefficient on the horizontal axis, taking exogenous correlation cost at $c = 0.1$ (the value that assures existence of our equilibrium for small correlation). The probability is increasing in the costs together with the share of cheaters. The indirect (through the evasion share) effect of correlation in the reports works in the same direction as the direct effect[3]. The indirect effect is conventionally explained by strategic interaction, the direct one makes sense, because with more correlation there is a higher chance for uncoordinated report to be cheating.

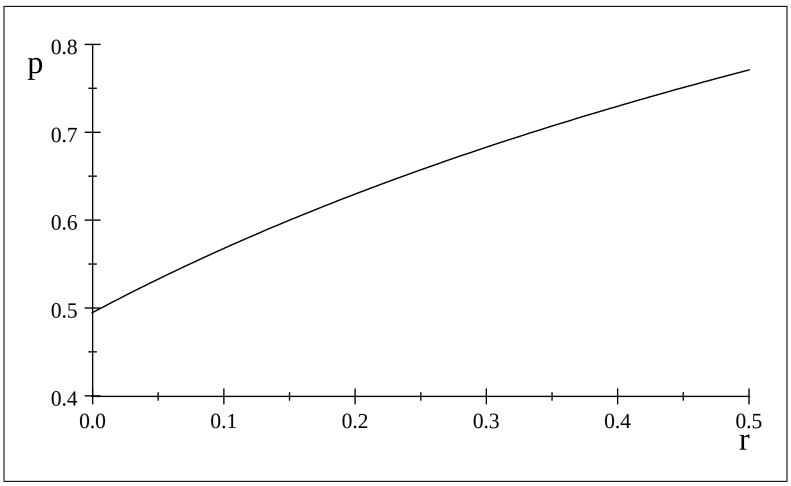

Figure 4. High low reports auditing probability $p\,(HL)$ depending on correlation r, low evasion regime, $c = 0.03$

This is the initial low cheating situation. The auditing probability is lower than in previous case and increasing. The values for correlation coefficient exceeding 50% are not precise, as the nonnegativity constraint of the auditing probability for coordinated reports is binding. The direct and indirect effects of the reports correlation act in opposite directions, and it turns out that the direct effect prevails.

[3]We can see clearly that the direct effect is positive, if we rewrite the auditing probability as $1 - k\frac{q+\frac{\gamma}{\delta}-1}{q(1+s)t\pi}$

3.3.3 Auditing probability - two low reports

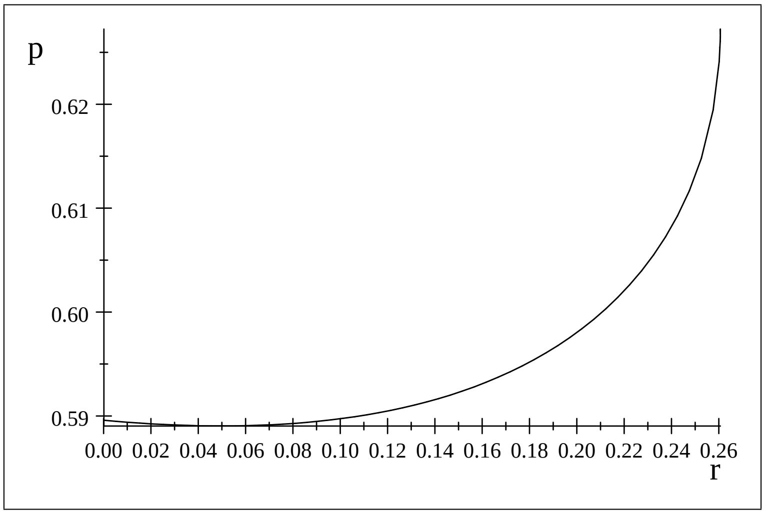

Figure 5. Two low reports auditing probability $p\,(HL)$ depending on correlation r, high evasion regime, $c = 0.1$

Here is the auditing of coordinated reports when the cheating is popular. It is not monotonous in the correlation coefficient, but is increasing on the most of the domain. Again the indirect effect is of course to enhance auditing, and the direct one to lower it[4]. Depending on which effect overtakes, we observe increasing or decreasing auditing probability.

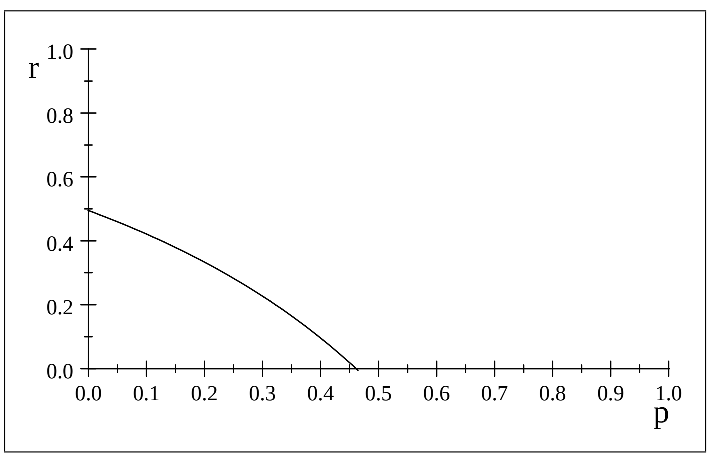

Figure 6. Two low reports auditing probability $p\,(HL)$ depending on correlation r, low evasion regime, $c = 0.1$

When the cheating is not popular, the probability to audit coordinated report is decreasing up to zero. Both direct and indirect effects work in the same direction,

[4]It can be shown that the derivative of the probability to audit coordinated reports wrt miscoordination costs has the same sign as $-q\,(1-q)\,(1+q\gamma)$.

inhibiting the auditing.

The stylized examples above nicely illustrate different policies towards miscoordination costs appropriate for different countries. The low auditing costs situation is more likely in developed countries with low level of evasion. In such cases the efforts to decrease miscoordination costs can be dangerous in a sense of bringing about more cheating. Moreover, this is coupled with more auditing, which is wasteful, as it does not reduce cheating. The high auditing costs picture is for the countries with flourishing evasion, like most of developing countries and CIS countries. These countries should not pay too much attention to correlation of profits, as increasing the costs of coordination may result in even larger cheating.

From this prospective, the Sorbanes-Oxley act can be justified as increasing costs c in the US. Unwillingness of many developing countries to be involved in a detailed analysis of industry structures in order to deduce true tax income can also be rationalized with the help of our model. This is certainly not to say that there are no more important factors underlying both phenomena, but simply to show that our model seems to go well with some stylized facts we know.

Tax revenue

Finally, we can see how the tax revenue is changing with correlation coefficient.

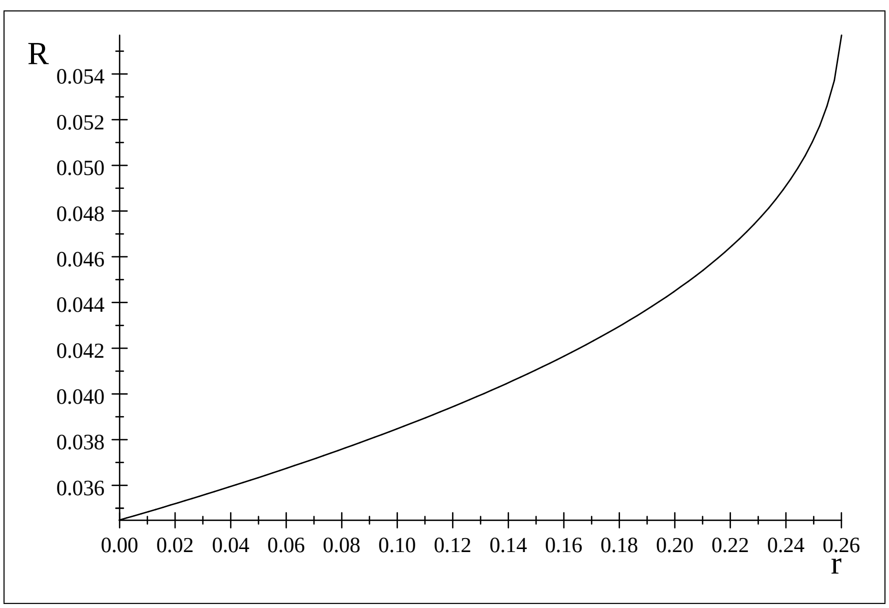

Figure 7. Tax revenue R depending on correlation r, high evasion regime, $c = 0.1$

Again, we have correlation coefficient on the horizontal axis. The revenue is increasing for both high evasion (above) and low evasion (below) regimes. This is not as intuitive as the decreasing after-tax income, because the auditors do not suffer miscoordination costs directly. The result is due to the interaction of various forces, but we see that the authority is able to use higher correlation of reports to enhance its revenue.

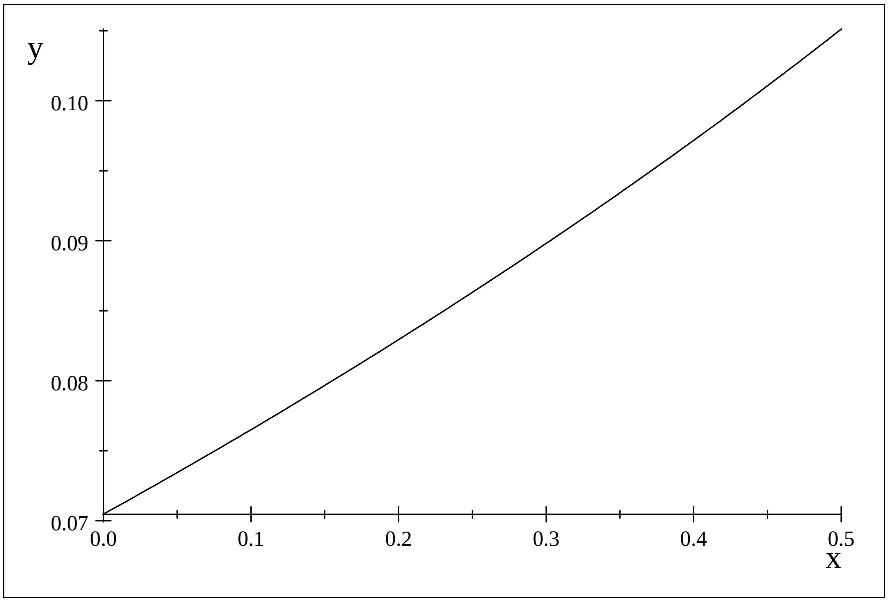

Figure 8. Tax revenue R depending on correlation r, low evasion regime, $c = 0.1$

4 Conclusion

The tax evasion game with costs of not coordinating decision between contracting firms is considered in this paper. We show that when miscoordination costs are small, there is a unique mixed strategy equilibrium with a positive share of evading firms and a positive share of audited reports. When the costs are substantial, there is a unique pure strategy equilibrium with either everybody or nobody evading, depending on the auditing effectiveness of the tax authority. If the miscoordination costs are very big, both full honesty and full cheating are equilibria, with an unstable mixture between them.

The game yields the insights that are impossible to obtain within the representative firm framework. Firstly, the tax authority should put more effort in auditing firms that did not coordinate their evasion decision, if it maximizes its expected revenue. Secondly, the coordination costs affect the amount of evasion in the opposite directions depending on what is the equilibrium share of cheating. If there are more then half non-compliant high income taxpayers, the coordination costs increase evasion, and visa versa. The correlation of taxpayer income acts in the same way as miscoordination costs. Finally, the situation when everybody honest is more likely to result the stronger are the links between taxpayers, that is the higher is correlation between their profits.

Another set of results is derived from the effects our parameters have on equilibrium cheating. We see that various costs considered here (two types of miscoordination costs and auditing costs) always reinforce each other's effect on equilibrium cheating. This complementarity calls for cost changing interventions, as reduction even in one type of costs will have an extended effect through other types. The fine effectiveness in our setup is influenced by miscoordination costs as well. It is affected positively when cheating is popular, and negatively in the opposite case. Finally, when almost everybody is evading, more high income taxpayers increase the share of cheating; when almost everybody is honest, the opposite is true.

There is a number of policy recommendations arising from our analysis. Firstly, miscoordination costs reduction efforts are only justified for economies (or industries) with substantial shadow sector. Such efforts include simplified accounting (exogenous costs) and little interest in the business links (endogenous costs through auditing probability differential). This, of course, is not applicable when the sole goal of the government is the tax authority's revenue, which is increasing in costs for our parameter values. Secondly, the miscoordination costs manipulation appears to be a promising way for achieving socially beneficial equilibria, as it works as a complement with other costs. Thirdly, fines become more efficient and therefore should be more widely used in low cheating - low miscoordination costs and high cheating - high miscoordination costs.

We hope that our paper opens up a whole tile of issues that could not be addressed by the literature before. How do the links between taxpayers affect their decision to pay taxes? How are these links taken into account by the tax authority? Could the government change the structure of these links for the benefit of the whole society? We can not answer these questions in a far too simplified setting of business pairs we have here. However, what we can do is to say that the equilibrium behavior of the agents is affected significantly by the links between them, that it is affected through the costs of behaving differently, and it is affected in the direction of harmonization of this behavior.

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Appendices

A - Proof of Lemma 1

The expected revenue of the auditor is

$$\begin{aligned}
&2\delta(1-q)^2 t\pi + (2\delta q(1-q) + 2(\gamma-\delta)(1-q)) \\
&* \left(t\pi + (1+s)\,p(HL)\frac{2\delta q(1-q)}{2\delta q(1-q) + 2(\gamma-\delta)(1-q)}t\pi - \hat{a}(HL)\right) \\
&+ \left(\delta q^2 + 2q(\gamma-\delta) + 1 - 2\gamma + \delta\right) \\
&* \left((1+s)\,t\pi p(LL)\frac{2\delta q^2 + 2q(\gamma-\delta)}{2\delta q^2 + 2q(\gamma-\delta) + 1 - 2\gamma + \delta} - 2a(LL)\right)
\end{aligned} \tag{7}$$

Rearranging and taking first order conditions with respect to $a(LL)$ and $\hat{a}(HL)$ gives

$$\begin{aligned}
\hat{a}(HL) &: \; -(2\delta q(1-q) + 2(\gamma-\delta)(1-q)) + 2\delta q(1-q)(1+s)\,p'(HL)\,t\pi = 0 \\
a(LL) &: \; \left(2\delta q^2 + 2q(\gamma-\delta)\right)(1+s)\,t\pi p'(LL) - 2\left(\delta q^2 + 2q(\gamma-\delta) + 1 - 2\gamma + \delta\right) = 0
\end{aligned}$$

Working this out, we arrive at

$$\begin{aligned}
p'(HL) &= \frac{\delta q + \gamma - \delta}{\delta q(1+s)\,t\pi} \\
p'(LL) &= \frac{\delta q^2 + 2q(\gamma-\delta) + 1 - 2\gamma + \delta}{(\delta q^2 + q(\gamma-\delta))(1+s)\,t\pi}
\end{aligned}$$

from which we can compare $a(LL)$ and $\hat{a}(HL)$. If $\hat{a}(HL) > a(LL)$, due to convexity of effort we have $p'(HL) < p'(LL)$, or

$$\frac{\delta q + \gamma - \delta}{\delta} < \frac{\delta q^2 + 2q(\gamma-\delta) + 1 - 2\gamma + \delta}{\delta q + \gamma - \delta}$$

since the infimum of the rhs denominator iz zero, we can muliply by it, to get after rearrangement

$$\gamma^2 < \delta$$

which is true for any positive correlation and holds with equality for independent draws.

To complete the lemma, we use the functional form of the probability

$$e^{-\frac{\hat{a}}{k}} = kp'(a)$$

to obtain the expressions (**??**) and (3) for the auditing effort.

B - Proof of proposition 2

Mixed strategies

To show that p, q is indeed a symmetric Bayesian Nash equilibrium, we need 1) p is a best response of tax authority given the belief about q; 2) q is a best response of each firm to the authority playing p and the other firm playing q; 3) the belief of the authority is consistent with equilibrium play of the firms.

For 1) we need (**??**) and (3); for 2) in a mixed equilibrium it is sufficient that each firm is indifferent between cheating and honesty given that the other high income firm is cheating with probability q:

$$\frac{\delta q+\gamma-\delta}{\gamma}\left(\pi-p\left(LL\right)\left(1+s\right)t\pi\right)+\frac{\delta}{\gamma}\left(1-q\right)\left(\pi-p\left(HL\right)\left(1+s\right)t\pi-c\right)=$$
$$=\frac{\delta}{\gamma}q\left(\pi\left(1-t\right)-c\right)+\left(1-\frac{\delta}{\gamma}q\right)\left(\pi\left(1-t\right)\right)$$

where we assume that there is no coordination costs of meeting the low income firm. Once one firm knows it has high profit, the conditional probabilities for the other firm to be high (low) are $\frac{\delta}{\gamma}(\frac{\gamma-\delta}{\gamma})$. Rearranging, we get

$$\left(\left(\delta q+\gamma-\delta\right)p\left(LL\right)+\delta\left(1-q\right)p\left(HL\right)\right)\left(1+s\right)t\pi=\gamma t\pi-\delta\left(1-2q\right)c$$

Substituting for the best response of the authority and rearranging, we have

$$2q^2\delta c+q\left(k\gamma-\gamma st\pi-\delta c\right)+k\left(1-\gamma\right)=0$$

Solving the quadratic equation and picking up the relevant root (the one that stays in the unit interval for reasonable parameter values), we get (6).

Of course, both auditing probabilities should satisfy probability restrictions. In our case it is sufficient to check that $p\left(HL\right)\leq 1$ and $p\left(LL\right)\geq 0$. The former is automatically satisfied, the latter is equivalent to

$$k\left(q\left(\gamma-\delta\right)+1-2\gamma+\delta\right)\leq\left(\delta q^2+q\left(\gamma-\delta\right)\right)\left(\left(1+s\right)t\pi-k\right)$$

which is insured for relatively small auditing costs and is in general more likely to hold for small correlation coefficients (and hence small δ). When this nonnegativity condition is violated, the robability to cheat is changed according to constraint optimization. The precise formulae can be derived in a similar fashion; we do not do it here because it is out of our focus.

If exogenous coordination costs are absent for the honest firms, we have

$$\frac{\delta q+\gamma-\delta}{\gamma}\left(\pi-p\left(LL\right)\left(1+s\right)t\pi\right)+\frac{\delta}{\gamma}\left(1-q\right)\left(\pi-p\left(HL\right)\left(1+s\right)t\pi-c\right)=\pi\left(1-t\right)$$

$$\frac{\delta q+\gamma-\delta}{\gamma}\left(-p\left(LL\right)\left(1+s\right)t\pi\right)+\frac{\delta}{\gamma}\left(1-q\right)\left(-p\left(HL\right)\left(1+s\right)t\pi-c\right)=-t\pi$$

$$\left(\left(\delta q+\gamma-\delta\right)p\left(LL\right)+\delta\left(1-q\right)p\left(HL\right)\right)\left(1+s\right)t\pi=\gamma t\pi-\delta\left(1-q\right)c$$

$$\left(\left(\delta q+\gamma-\delta\right)\left(1-k\frac{\delta q^2+2q\left(\gamma-\delta\right)+1-2\gamma+\delta}{\left(\delta q^2+q\left(\gamma-\delta\right)\right)\left(1+s\right)t\pi}\right)+\delta\left(1-q\right)\left(1-k\frac{\delta q+\gamma-\delta}{\delta q\left(1+s\right)t\pi}\right)\right)\left(1+s\right)t\pi=\gamma t\pi-$$

$$\left(\delta q+\gamma-\delta\right)\left(1+s\right)t\pi-k\frac{\delta q^2+2q\left(\gamma-\delta\right)+1-2\gamma+\delta}{q}+\delta\left(1-q\right)\left(\left(1+s\right)t\pi-k\frac{\delta q+\gamma-\delta}{\delta q}\right)=\gamma t\pi-\delta$$

$$\gamma\left(1+s\right)t\pi-k\frac{\delta q^2+2q\left(\gamma-\delta\right)+1-2\gamma+\delta}{q}-k\frac{\delta q+\gamma-\delta}{\delta q}\delta\left(1-q\right)=\gamma t\pi-\delta\left(1-q\right)c$$

$$\gamma st\pi-k\left(\frac{\delta q^2+2q\left(\gamma-\delta\right)+1-2\gamma+\delta}{q}+\frac{\delta q+\gamma-\delta-\delta q^2-q\left(\gamma-\delta\right)}{q}\right)=-\delta\left(1-q\right)c$$

$$\gamma st\pi-k\frac{q\gamma+1-\gamma}{q}=-\delta\left(1-q\right)c$$

$$\gamma st\pi q-k\left(q\gamma+1-\gamma\right)=-\delta\left(1-q\right)cq$$

$$q^2\delta c+q\left(k\gamma-\gamma st\pi-\delta c\right)+k\left(1-\gamma\right)=0$$

Pure strategies

Full cheating condition is

$$\pi-p\left(LL,q=1\right)\left(1+s\right)t\pi\quad>\quad\pi\left(1-t\right)$$

After substituting for the best response of authority with consistent belief about full cheating $p\left(LL,q=1\right)=1-\frac{k}{\gamma(1+s)t\pi}$ and simplifying, we arrive at

$$\delta c>\gamma st\pi-k \tag{8}$$

Full honesty condition

$$\frac{\gamma-\delta}{\gamma}\left(\pi-p\left(LL,q=0\right)\left(1+s\right)t\pi\right)+\frac{\delta}{\gamma}\left(\pi-p\left(HL,q=0\right)\left(1+s\right)t\pi-c\right)<\pi\left(1-t\right)$$

After substituting for the best response of authority with consistent belief about full honesty $p\left(LL\right)=p\left(HL\right)=0$, we get

$$\gamma t\pi<\delta c$$

Combining the results for mixed and pure strategies, we get the statement of the proposition.

C - payoffs computation

Representative The expected payoff of the authority in this case is by definition

$$R = (1-q)\,\gamma t\pi + (1-\gamma+q\gamma)\left(\frac{q\gamma}{1-\gamma+q\gamma}p\,(1+s)\,t\pi - a\right)$$

This can be rearranged to get

$$R = (1-q+pq\,(1+s))\,\gamma t\pi - (1-\gamma+q\gamma)\,a$$

or, substituting for equilibrium values of p and q,

$$R = t\pi\left(\gamma + (1-\gamma)\frac{sk}{st\pi - k}\ln\frac{s}{1+s}\right)$$

Income of the high type is by definition

$$I = q\,(\pi - p\,(1+s)\,t\pi) + (1-q)\,(1-t)\,\pi$$

which can be rearranged to

$$I = (1-t)\,\pi$$

Note that the same is obtained by the logic of indifference between cheating and staying honest.

Coordinated For the authority the expected revenue (7) can be represented as

$$\begin{aligned}
&R = 2\delta\,(1-q)^2\,t\pi + (2\delta q\,(1-q) + 2\,(\gamma-\delta)\,(1-q))\,(t\pi - \hat{a}\,(HL)) \\
&+ (1+s)\,p\,(HL)\,2\delta q\,(1-q)\,t\pi - 2a\,(LL)\left(\delta q^2 + 2q\,(\gamma-\delta) + 1 - 2\gamma + \delta\right) \\
&+ (1+s)\,t\pi p\,(LL)\left(2\delta q^2 + 2q\,(\gamma-\delta)\right)
\end{aligned} \tag{9}$$

which can be re-arranged to

$$\begin{aligned}
\frac{R}{2} &= \gamma\,(1-q)\,t\pi + q\gamma\,(1+s)\,t\pi - k\,(q\gamma + 1 - \gamma) \\
&\quad - (\delta q + \gamma - \delta)\,(1-q)\,\hat{a}\,(HL) - a\,(LL)\left(\delta q^2 + 2q\,(\gamma-\delta) + 1 - 2\gamma + \delta\right)
\end{aligned}$$

which is not incredibly intuitive, but outlines two sources of income and two sources of costs. The income consists of "voluntary" contributions by the honest and fines from the cheaters (net of foregone fines from non-caught cheaters). The costs are in efforts on auditing two types of reports. With independent incomes, the expression takes the form of

$$\frac{R}{2} = \gamma\,(1+qs)\,t\pi - (q\gamma + 1 - \gamma)\,(k+a)$$

note that this is the same as in the representative case, only the values for p, q, a are different.

Using the indifference conditions we can conveniently write income from honest behavior

$$I = \frac{\delta}{\gamma} q \left(\pi \left(1 - t\right) - c\right) + \left(1 - \frac{\delta}{\gamma} q\right) \left(\pi \left(1 - t\right)\right)$$

which is simplified to

$$I = \pi \left(1 - t\right) - \frac{\delta}{\gamma} qc$$

So, the expected income of the high income taxpayers is negatively related to the equilibrium cheating level and both exogenous and endogenous coordination costs.

D - derivation of the comparative statics results

Miscoordination costs and correlation coefficient

$$\begin{aligned} q^2 2\delta c + q\left(k\gamma - \gamma st\pi - \delta c\right) + k\left(1 - \gamma\right) &= 0 \\ 2\delta\left(q^2 dc + 2qcdq\right) + dq\left(k\gamma - \gamma st\pi - \delta c\right) - \delta q dc &= 0 \\ dq\left(\left(4q - 1\right)\delta c + \gamma\left(k - st\pi\right)\right) &= \delta q\left(1 - 2q\right) dc \end{aligned}$$

Let us consider few cases. First, if $q > \frac{1}{2}$: rhs is negative; lhs is also negative. Thus, $\frac{dq}{dc} > 0$. This makes sense. If $q < \frac{1}{2}$, rhs is positive and lhs is negative. thus, $\frac{dq}{dc} < 0$. This is a very nice result with simple intuition: when there are more than half of cheaters, the greater coordination costs increase equilibrium evasion, and visa versa.

The same is true for the correlation coefficient, that gives us the following expression

$$dq\left(4cq\delta + k\gamma - \gamma st\pi - \delta c\right) = cq\left(1 - 2q\right) d\delta$$

If under given parameters there are less than half cheating people, equilibrium evasion decreases in correlation increase. This is less intuitive, but note that the higher correlation acts exactly in the same way as the miscoordination costs. Actually, it is exactly part of miscoordination costs, which is endogenous in our model - a markup on auditing probability faced by miscoordinated reports.

Actually, we can show how the markup is increasing with correlation:

$$p\left(HL\right) - p\left(LL\right) = 1 - k\frac{\delta q + \gamma - \delta}{\delta q\left(1 + s\right) t\pi} - 1 + k\frac{\delta q^2 + 2q\left(\gamma - \delta\right) + 1 - 2\gamma + \delta}{\left(\delta q^2 + q\left(\gamma - \delta\right)\right)\left(1 + s\right) t\pi}$$

which can be rearranged to

$$\frac{k}{q(1+s)t\pi}\left(\frac{1-4\gamma(1-q)-\frac{\gamma^2}{\delta}}{\delta(q-1)+\gamma}\right)$$

From here it is visible that the markup is increasing with δ.

Second order costs

$$\begin{aligned}\frac{d^2q}{dc^2} &= -\frac{\delta q^2(1-2q)(4q-1)}{((4q-1)\delta c+\gamma(k-st\pi))^2}\\ \arg sign &= \delta q^2(1-2q)(1-4q)\end{aligned}$$

So, the cheating is convex in costs for intervals $\left(0,\frac{1}{4}\right)$ and $\left(\frac{1}{2},1\right)$ of the range; it is concave on the rest of the interval.

Cross effect

Another interesting thing is to see $\frac{dq}{dc}(r) = 0$:we do it in a separate file without apparent success.

$$\begin{aligned}dq(4cq\delta+k\gamma-\gamma st\pi-\delta c) &= qc(1-2q)d\delta\\ \frac{dq}{d\delta} &= \frac{qc(1-2q)}{((4q-1)\delta c+\gamma(k-st\pi))}\end{aligned}$$

differentiate (and define $S := st\pi - k$)

$$\begin{aligned}\frac{d^2q}{d\delta dc} &= \frac{\left((1-4q)\frac{dq}{dc}c+q(1-2q)\right)((4q-1)\delta c+\gamma(k-st\pi))-qc(1-2q)\delta\left(4\frac{dq}{dc}c+4q-1\right)}{((4q-1)\delta c+\gamma(k-st\pi))^2}\\ &\quad \frac{\left((1-4q)\frac{dq}{dc}c+q(1-2q)\right)((4q-1)\delta c-\gamma S)-qc(1-2q)\delta\left(4\frac{dq}{dc}c+4q-1\right)}{((4q-1)\delta c-\gamma S)^2}\\ c &= 0:\frac{q(1-2q)(-\gamma S)}{(-\gamma S)^2}=\frac{-q(1-2q)}{\gamma S}\end{aligned}$$

so, the initial cross effect is negative for the decreasing part of the function ($q < \frac{1}{2}$); it is positive for the increasing part of the funcion. Thus, costs of different types reinforce each other at low exogenous costs, which is not counterintuitive, as there is no "congestion" effect, but only complementarity of costs. If cheating is popular ($q > \frac{1}{2}$), increasing of one type of the costs has the stronger cheating-boosting effect on equilibrium, the larger is the costs of the other type.

Fine

The fine is obviously expected to have a deterring effect on the evasion. Indeed, we see that

$$\begin{aligned} 4\delta cqdq + dq\left(k\gamma - \delta c\right) - \gamma t\pi\left(sdq + qds\right) &= 0 \\ \left(\left(4q-1\right)\delta c - \gamma S\right)dq &= qds \end{aligned}$$

More interestingly, the deterrence effect is unambiguously decreasing with the cheating popularity

$$\left(\frac{dq}{ds}\right)'_q = \frac{\left(4q-1\right)\delta c - \gamma S - 4q\delta c}{\left(\left(4q-1\right)\delta c - \gamma S\right)^2} = \frac{-\delta c - \gamma S}{\left(\left(4q-1\right)\delta c - \gamma S\right)^2}$$

Thus, initial state matters a lot in our game: it does not only determine the effect of the miscoordination costs on the cheating, but also the extent to which fine can be effective in deterring evasion. The effect of the costs on the deterrence effectiveness is also of interest:

$$\begin{aligned} \frac{d^2q}{dsdc} &= \frac{q'_c\left(\left(4q-1\right)\delta c - \gamma S\right) - q\left(4q'_c\delta c + \left(4q-1\right)\delta\right)}{\left(\left(4q-1\right)\delta c - \gamma S\right)^2} \\ &\quad \frac{-q'_c\left(\delta c + \gamma S\right) - q\delta\left(4q-1\right)}{\left(\left(4q-1\right)\delta c - \gamma S\right)^2} \end{aligned}$$

The denominator is obviously positive. The first term in the nominator is positive for $q < \frac{1}{2}$ and negative in complementary case; the second term is positive for $q < \frac{1}{4}$ and negative in complementary case. So, for high cheating the costs increase the effectiveness of fines. For low cheating the opposite is true, whereas for $\frac{1}{4} < q < \frac{1}{2}$ the effect is ambiguous. Well, at least we see that surprisingly enough, the costs are incremental for the fee effectiveness when cheating is popular.

Finally, we can see how the effectiveness of the fine changes with its own amount:

$$\begin{aligned} \frac{d^2q}{ds^2} &= \frac{q'_s\left(\left(4q-1\right)\delta c - \gamma S\right) - q\left(4q'_s\delta c - \gamma t\pi\right)}{\left(\left(4q-1\right)\delta c - \gamma S\right)^2} \\ &\quad \frac{-q'_s\left(\delta c + \gamma S\right) + q\gamma t\pi}{\left(\left(4q-1\right)\delta c - \gamma S\right)^2} \end{aligned}$$

Again, with small cheating the first term is positive; it becomes negative in the opposite case. The second term is always positive, increasing in the evasion share. If the following condition is true, we then have cheating being convex in the fine (thus, every increase is less and less effective):

$$\left|\left(4q-1\right)\delta c - \gamma S\right| < 1$$

this condition is though not guaranteed by anything, so we do not have pure convexity result.

Auditing costs

We expect auditing costs to unambiguously favor evasion. Differentiating, we get

$$dq\left(\left(1-4q\right)\delta c+\gamma S\right)=dk\left(1-\gamma+q\gamma\right)$$

which is positive indeed. The interesting thing is to see how our costs influence the auditing costs effect on cheating:

$$\frac{d^2q}{dkdc}=\frac{q_c'\left(\left(1-4q\right)\delta c+\gamma S\right)-\left(1-\gamma+q\gamma\right)\delta\left(-4q_c'c+1-4q\right)}{\left(\left(1-4q\right)\delta c+\gamma S\right)^2}$$

does not look very intuitive. The other way is to differentiate the coordination costs effect:

$$\begin{aligned}\frac{dq}{dc} &= \frac{\delta q\left(1-2q\right)}{\left(4q-1\right)\delta c+\gamma\left(k-st\pi\right)}\\ \frac{d^2q}{dcdk} &= \frac{\delta q_k'\left(1-4q\right)\left(\left(4q-1\right)\delta c+\gamma\left(k-st\pi\right)\right)-\delta q\left(1-2q\right)\left(4q_k'\delta c+\gamma\right)}{\left(\left(4q-1\right)\delta c+\gamma\left(k-st\pi\right)\right)^2}\end{aligned}$$

also not very nice. Still, the first term is negative whenever $q<\frac{1}{4}$ and positive otherwise. The second term is also negative whenever $q<\frac{1}{2}$. The derivative is then unambiguosly negative for $q<\frac{1}{4}$, and unambiguosly positive for $q>\frac{1}{2}$. On the rest of the unit interval the parameter combinations determine the sign. But the general tendency is clear: the two types of costs are reinforcing each other, just like in the other case. To this instance, we would also like to look at the interaction of the source of endogenous costs (correlation) with auditing costs. We are likely to get the same since the expressions are so similar. We shall get the following kind of matrix for our interactions:

	c	δ	k
c	$+$		
δ	$+$	$+$	
k	$+$	$+$	$+$

As for the fine, the cross derivative is

$$\frac{d^2q}{dkds}=\frac{q_s'\left(\left(1-4q\right)\delta c+\gamma S\right)+\left(1-\gamma+q\gamma\right)\left(4q_s'c\delta+\gamma t\pi\right)}{\left(\left(1-4q\right)\delta c+\gamma S\right)^2}$$

which i am also not really happy about. But the first term seems to be negative, whereas the second is ambiguous. The whole thing is negative, if

$$q + (1 - \gamma + q\gamma)\left(4q_s' c\delta + \gamma t\pi\right) < 0$$

the question is how important it is at all.

Share of high income taxpayers

Differentiating the initial condition wrt gamma we get

$$\begin{aligned} \left(4q\delta c + k\gamma - \gamma st\pi - \delta c\right) dq + \left((q-1)k - qst\pi\right) d\gamma &= 0 \\ \frac{dq}{d\gamma} &= \frac{qS + k}{(4q-1)\,\delta c - \gamma S} \end{aligned}$$

Obviously, this is positive for small enough γ and large enough q, namely $(4q-1)\,\delta c > \gamma S$. For small $q < \frac{1}{4}$ this is definitely negative, so for prevalent honesty increasing share of high incomes brings about even more honesty. For large cheating the same effect is more likely to result even in more cheating.

Income differential and tax rate

Income differential in our setup is represented by parameter π as low income level is normalized to zero. As the tax rate t, the income differential does not enter our equations separately from the surcharge rate, so their effect has the same sign as the fine s.

Auditing probability

We know that the mixed equilibrium auditing probability is increasing in the share of cheaters when reports are the same, and increasing for some parameter values () when the reports are different. This result can be used in the comparative statics derivation for the auditing probabilities. First, consider the auditing probability p of auditing LL report:

$$p = 1 - \frac{k}{(1+s)\,t\pi}\left(\frac{1-\gamma}{q\gamma} + 1\right)$$

Coordination costs enter this expression only through q, so we have the result mirroring the previuos one: when the cheating is not popular, the coordination costs decrease the auditing probability p. Deterrence variables (surcharge rate, tax rate and income differential) act in the opposite direction through their direct (increasing auditing) and indirect (decreasing cheating) channels. Namely,

$$
\begin{aligned}
-\frac{dp}{ds} &= -\frac{k}{(1+s)^2\, t\pi}\left(\frac{1-\gamma}{q\gamma}+1\right) - \frac{k}{(1+s)\, t\pi}\frac{1-\gamma}{q^2 \frac{dq}{ds}\gamma} \\
\frac{dp}{ds} &= \frac{k}{(1+s)\, t\pi}\left(\frac{1}{1+s}\left(\frac{1-\gamma}{q\gamma}+1\right) + \frac{1-\gamma}{q^2 \frac{dq}{ds}\gamma}\right)
\end{aligned}
$$

Thus, stricter enforcement actually raises zeal of tax inspectors whenever

$$
\begin{aligned}
\frac{1}{1+s}\left(\frac{1-\gamma}{q\gamma}+1\right) &> -\frac{1-\gamma}{q^2 \frac{dq}{ds}\gamma} \\
1+\frac{q\gamma}{1-\gamma} &> -\frac{1+s}{q\frac{dq}{ds}}
\end{aligned}
$$

which is more likely to hold when cheating is popular. A similar story happens with the auditing costs k: they increase evasion, but also increase auditing directly. In sum,

$$
-\frac{dp}{dk} = \frac{1}{(1+s)\, t\pi}\left(\frac{1-\gamma}{q\gamma}+1\right) - \frac{k}{(1+s)\, t\pi}\frac{1-\gamma}{q^2 \frac{dq}{dk}\gamma}
$$

And the auditing is more intensive with its own costs if

$$
1-\gamma+q\gamma < k\frac{1-\gamma}{q\frac{dq}{dk}}
$$

here we go one step further and plug in the actual expression for the derivative to get

$$
(1-\gamma+q\gamma)^2 < k\frac{1-\gamma}{q}\left((1-4q)\,\delta c+\gamma S\right)
$$

which is a reasonable expression, hehe.

The last thing to worry about is γ, but since we did not get any clear-cut result before, we do not expect miracles here:

$$
\frac{dp}{d\gamma} = -\frac{k}{(1+s)\, t\pi}\frac{-q\gamma-(1-\gamma)(q_\gamma\gamma+q)}{(q\gamma)^2}
$$

As q_γ is likely to be negative, we have again direct and indirect effects working in opposite directions. When the direct effect overweighs, the total derivative is positive, so that with increaing share of high income taxpayers the auditing probability is increasing.