Fertility, economic growth and welfare in an OLG model with regulated wages

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Abstract We analyse the effects of the regulation of wages in a standard neoclassical OLG growth model extended to account for endogenous fertility. In contrast with the prevailing literature, which has failed to pay due attention to inter-temporal contexts, our conclusion is that under suitable conditions – that is, sufficiently high capital's weight in technology and unemployment benefits – a regulated wage economy may perform better than a market-wage economy as regards both the long-run economic growth and the lifetime welfare of the representative generation. As a consequence, the correlation between unemployment and economic growth may be positive. Further, the regulated wage may also be treated as a policy parameter for the control of population growth.

Keywords Endogenous Fertility; Regulated Wage; OLG Model; Welfare

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1 Introduction

Although a vast debate about the macroeconomic consequences of minimum wages[1] has been developed dating from Stigler (1946), less attention has been paid to the long run effects of the regulation of wages in a dynamical context (i.e., a simple OLG framework). The idea that the introduction of minimum wages in a simple competitive economy always brings an output loss due to unemployment is a widespread belief.[2] Furthermore, the prevailing literature (e.g., among others, Bean and Pissarides, 1993; Daveri and Tabellini, 2000) establishes a negative correlation between unemployment and growth: i.e., unemployment always deteriorates growth.

Indeed only few papers have found possible positive long run macroeconomic effects of the minimum wage legislation in an inter-temporal OLG context, for example Cahuc and Michel (1996) and Ravn and Sorensen (1999). However, in these papers such a possibility depends on the specific assumption of a positive relationship between the unemployment created by the minimum wage and the long-run productivity growth induced by schooling and by training on the job; such a disputable assumption introduced a positive externality which implies a departure from the conventional OLG model (Diamond, 1965) used and we have adopted. In particular, none of these models are concerned with the role played by government interventions on the labour market such as the minimum wage legislation accompanied by unemployment insurance, in a context of endogenous fertility. In this paper we will try to fill the gap by developing a standard neoclassical OLG growth model embodying such features. The value added of this paper grounds on three novel results, which have so far escaped closer scrutiny: i) under suitable conditions – that is, sufficiently high weight for capital in technology and unemployment benefits – a regulated wage economy may perform better than a market-wage economy in the long-run; ii) the correlation between unemployment and economic growth may be positive; and iii) the level of the regulated wage may also be treated as a policy parameter for controlling population growth.

The basic idea behind our results may be summarised as follows: a labour market imperfection which gives rise to a wage hike, will increase the income of the currently young generation on one side, but on the other side, it will lead to unemployment as the hourly wage is fixed at too high a level for the labour market to be cleared. In the short-run, this brings about a decrease in the overall income of the younger generation (despite the presence of unemployment insurance benefits) and, given the constant propensity to save, a decrease in savings as well. However, the introduction of minimum wages establishes a (short-run) negative correlation between unemployment and fertility, and, in particular, fertility reduces more than savings. In the light of the latter effect, the pace of accumulation of capital per-capita (which depends on the ratio between savings and the demand for children) in the regulated-wage economy is higher than the capital accumulation path in the competitive-wage economy. The possible increase in capital accumulation is, however, only a necessary engine of a possible higher economic growth.[3] Another condition – which depends on the technology – is necessary for the long-run income to be improved by the regulation of wages, and the mechanism is the following: in contrast with the static context in which a regulated wage always brings upon an output loss due to a labour demand reduction, the other input (the capital stock) being fixed, in a dynamic context the effect of minimum wage legislation on economic growth depends on two counterbalancing forces: on the one hand the induced increase in the capital input

[1] It is worth noting that in this model where, for simplicity, only one type of labour does exist, a binding minimum wage simply indicates a regulated wage fixed by law over the market-clearing level. In the case of more than one type of labour with uniformly distributed wages, this assumption would simply mean a regulated wage fixed over the average market wage.

[2] For instance, "it is generally recognized that minimum wage legislation induces distortions which have adverse effects on the efficiency of the economy." (Cahuc and Michel, 1996, p. 1464).

[3] In this paper the term economic growth always refers to the level (rather than to the rate of growth) of the long run income, according to the terminology of the neoclassical growth theory (e.g., Solow, 1956; Mankiw et al., 1992). In any case, needless to say, an increase in the long run level of output, implies a transitional increase in the rate of growth as well.

and on the other hand the induced reduction in the labour input. Whether (transitional) economic growth and long-run output will be reduced or increased by the introduction of a regulated wage will depend ultimately on the weight of the capital input in the production function.

Therefore, if capital accumulation is improved by the regulation of wages and the capital's weight in production is sufficiently high, the long-run economic growth, that is the transitional rate of growth as well as the long run level of per-capita income, is higher in a regulated-wage regime than in the standard competitive-wage economy. But the noteworthy improvement in economic growth is not the end of the story: the regulation of wages may also generate an increase of the "felicity" due to the increased lifetime consumption which overcompensates the reduction in the number of children in the welfare evaluation, thus also enhancing the lifetime welfare, provided that individuals are not too interested in smoothing consumption over time and having children.[4] Finally we note that, when economic growth is enhanced by the wage regulation, a self-evident corollary is that the correlation between unemployment and economic growth, in contrast with the prevailing literature, may be positive.[5]

It must be emphasised that we have sought to clarify these theoretical findings using as parsimonious a model as possible, that is the standard Diamond's (1965) OLG model of growth extended to account for endogenous fertility and minimum wage legislation.

The paper is organised as follows. In section 2 we present the model and the main results are analysed and discussed by comparing both competitive and the regulated wage economies, whilst in section 3 a graphical illustration of the results is shown. Finally, section 4 bears the conclusions.

2 The model

We characterise a basic dynamic general equilibrium OLG growth model[6] – as in Diamond (1965) – extended to account for endogenous fertility and regulated wages. The economy is closed to international trade, and goods and capital markets are both competitive. The model is outlined as follows.

2.1 Individuals. Agents have identical preferences and are assumed to belong to an overlapping generations structure with finite lifetimes. Life is separated among three periods: childhood, young adulthood and old-age. During childhood individuals do not make decisions. Adult individuals belonging to generation, say, t have a homothetic and separable utility function defined over consumption when young ($c_t^{\,y}$) and old ($c_{t+1}^{\,o}$) and from having children (n_t),[7] as in Galor and Weil (1996).[8] Only young-adult individuals (N_t) join the workforce assuming a unitary constant labour supply. As an adult each agent earns a constant binding minimum wage ($\underline{w}$) fixed by law over the prevailing market-clearing level: thus, the labour market does not clear and involuntary

[4] We note that whether the regulation of wages is both growth and welfare enhancing or not depends ultimately only on tastes, technology and the level of the replacement ratio.

[5] The positive effect of unemployment on both economic growth and welfare, as shown in this paper, does not exclusively depend either on the type of taxation or on the hypothesis of endogenous fertility. For instance it may hold even with exogenous fertility and with different tax policies (e.g., consumption taxes, capital income taxes and so on), evidencing that the results of this paper are a robust feature of regulated wage economies (see Fanti and Gori, 2007).

[6] Two reference textbooks are Azariadis (1993) and De La Croix and Michel (2002).

[7] Note that n_t represents the number of children with $n_t - 1$ being the population growth rate (for simplicity, the mortality rate has not been included in the analysis). Some authors, including Samuelson (1975), used $N_{t+1}/N_t = 1+n$ with n representing the population growth rate. Our approach is used in most papers with endogenous fertility.

[8] Since one scope of this paper is to isolate the relation among individuals' fertility and regulated wages, as a first attempt we ignore both the trade-off between child quantity and quality and the assumption that parents maximise the utility of their offspring, which have been employed to explain economic growth and stagnation by – among others – Becker et al. (1990) and Ehrlich and Lui (1991).

unemployment occurs. In order to tackle the unemployment issue, we assume the existence of an unemployment insurance mechanism which pays proportional-to-wage benefits for the unemployed time, i.e. $b(\underline{w}) := \gamma \underline{w}$, with $0 < \gamma < 1$ being the so-called replacement ratio, represents the hourly unemployment benefit perceived by each young-adult individual.[9] The aggregate unemployment rate (defined in terms of hours not worked) is $u_t = (N_t - L_t)/N_t$, where L_t is the labour demand. Thus, the representative individual's total income – as given by the sum of the working income plus the unemployment benefit – is $W_t := \underline{w}(1-u_t) + \gamma \underline{w} u_t$.[10] This income is used to consume, to raise children and to save. Further, we suppose that rearing children requires a monetary cost indexed with the total worker's income, that is the cost of having one child is simply $q \cdot W_t$ with $0 < q < 1$.[11] During old-age agents are retired and live on the proceeds of their savings (s_t) plus the accrued interest at the rate r_{t+1}.

The representative individual born at time t is faced with the problem of maximising the following logarithmic utility function:

$$\max_{\{c_t^y, c_{t+1}^o, n_t\}} U_t\left(c_t^y, c_{t+1}^o, n_t\right) = (1-\phi)\ln\left(c_t^y\right) + \phi \ln\left(c_{t+1}^o\right) + \rho \ln(n_t),$$

subject to inter-temporal budget constraint

$$c_t^y + c_{t+1}^o/(1+r_{t+1}) = W_t \cdot (1 - qn_t) - \tau_t,$$

where $\tau_t > 0$ is a lump-sum tax on the young-adult generation, $0 < \phi < 1$ represents a consumption preference parameter, with $\phi/(1-\phi)$ being the subjective discount rate and $\rho > 0$ captures the weight of the parents' taste for children in the welfare evaluation. The higher ϕ the more individuals smooth consumption over time, and the higher ρ the more parents are children-interested.

The first order conditions for an interior solution are:

$$\frac{c_{t+1}^o}{c_t^y} = \frac{\phi}{1-\phi}(1+r_{t+1}), \tag{1}$$

$$\frac{\rho}{n_t} = \frac{1-\phi}{c_t^y} \cdot qW_t. \tag{2}$$

[9] This is the typical case of the Italian unemployment insurance system (i.e., *Cassa Integrazione Guadagni*) which pays benefits for the unemployed hours due to temporary and partial layoffs.

[10] Note that in this model there is no uncertainty. Thus, each young-adult agent will be employed for $1-u_t$ hours and unemployed for u_t hours. We also assume that unemployed hours have no economic value. The use of the unemployed time either for self-enrichment activities or for exploiting home production technologies are interesting extensions which are beyond the scope of the present paper.

[11] This assumption is rather usual in literature (e.g., Wigger, 1999; Strulik, 2004; Boldrin and Jones, 2002; Fanti and Manfredi, 2003). For a better understanding of such an assumption, the cost of children may be interpreted as money spent, for instance, either for babysitting activities or for other specialised children services (for which parents must pay a gross-of-taxes price). This means that an increase in wages involves the parents' income as well as, for example, the babysitters' wage and therefore it implies that an increase in the parents' gross-of- tax income induces a proportional increase in the cost to raising children, with the consequence that fertility rates will be dramatically reduced when the gross-of-tax income increases. This hypothesis may be conveniently addressed in the model by postulating the cost of children to be a fraction of the gross-of-taxes income. Moreover, this assumption could also be interpreted as a proxy of the opportunity cost of the parents' home time which is increasing in their working income (see Cigno, 1991). However, if, alternatively, it were supposed that the "standard of living" of each child is a percentage of the parents' one (that is less disposable parents' income implies less childcare expenditure), then the right-hand side of the inter-temporal budget constraint would become $(W_t - \tau_t) \cdot (1 - qn_t)$, so that the government intervention would have no effects on both macroeconomic and demographic variables.

Eq. (1) equates the marginal utility of current and future consumption in terms of current consumption, whereas eq. (2) equates the marginal utility of having a child (ρ / n_t) with the involved marginal costs in terms of forgone utility of consumption ($(1-\rho)/c_t^{\,y}$).
Using eqs. (1) and (2) along with the inter-temporal budget constraint we get:

$$n_t = \frac{\rho}{1+\rho} \cdot \frac{W_t - \tau_t}{qW_t}, \tag{3}$$

$$s_t = \frac{\phi}{1+\rho}\left(W_t - \tau_t\right). \tag{4}$$

2.2 Government. One effect of the regulation of wages is to cause a positive level of unemployment. Further, the presence of an unemployment benefit mechanism raises the need to finance the payment of benefits. We suppose the government levies and adjusts over time lump-sum taxes on the younger generation such as to balance out unemployment benefit expenditures with tax receipts in every period. Thus, the time-t government constraint is:[12]

$$\gamma \underline{w} u_t = \tau_t . \tag{5}$$

An important feature of this taxation policy should be pointed out here: we have deliberately chosen a purely redistributive tax policy in which only the younger generation is involved, that is income taxed away from the young returned to the same individuals as a benefit for the hours of unemployment. This distinctive feature is important because in OLG models, as known dating back to Atkinson and Sadmo (1980), Sinn (1987), Bertola (1996) and Uhlig and Yanagawa (1996), either taxes on capital income or any other form of transfer from the old to the young could lead to faster economic growth. Therefore, since in our model taxation policy does not cause any transfer from the old-age to the younger generation (as, instead, it would have been the case with capital income taxes), the effects on demographic and macroeconomic variables as well as on the lifetime welfare of the representative generation – as described in this paper – should be entirely ascribed to the working of both minimum wages and unemployment benefits rather than to the intergenerational tax transfer channel.

2.3 The Number of Children and the Savings Path. Inserting (5) into (3) and (4) to eliminate τ_t, the number of children and the savings path are determined by:

$$n_t = \frac{\rho}{1+\rho} \cdot \frac{1-u_t}{q\left[1-u_t(1-\gamma)\right]}, \tag{6}$$

$$s_t = \frac{\phi}{1+\rho}\underline{w}\left(1-u_t\right), \tag{7}$$

where $\phi/(1+\rho)$ is the (constant) propensity to save. As it can be easily ascertained by looking at eqs. (6) and (7), in the short-run population falls short monotonically as both the unemployment rate and the replacement ratio rise, whilst savings are enhanced by increasing $\underline{w}$ and reducing u_t, with s_t being independent of unemployment benefit policies.

2.4 Firms. There are two factors of production: physical capital (K) and labour (L). The representative firm owns a constant returns to scale Cobb-Douglas technology by which the inputs of production are transformed into final goods and services, that is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$,[13] where $A > 0$

[12] It is important to note that all the results obtained in this paper would be confirmed even if we assume that individuals are ultra-perfect foresighted, that is they know the government policy as well as the behaviour of all other agents (and thus they would be able to internalise the government's budget constraint).
[13] Adding exogenous growth in labour productivity does not alter any of the substantive conclusions of the model and, hence, it is not included here.

represents a scale parameter and $\alpha \in (0,1)$ is the capital's weight in technology. It hires aggregate capital stock as well as labour ($L_t = (1-u_t)N$) – according to their marginal productivity – to maximise profits. Defining $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ as capital and output per-capita respectively, the intensive form production function becomes:

$$y_t = A(1-u_t)(k_t/(1-u_t))^{\alpha}. \tag{8}$$

Assuming that final output is traded at a unit price, profits maximisation leads to the following marginal conditions for capital and labour respectively:[14]

$$r_t = \alpha A(k_t/(1-u_t))^{\alpha-1} - 1, \tag{9}$$

$$\underline{w} = (1-\alpha)A(k_t/(1-u_t))^{\alpha}. \tag{10}$$

As far as labour is concerned, the marginal product of labour will adjust to meet the fixed real wage with u_t being endogenously determined. Thus, exploiting eq. (10) the short-run rate of unemployment is given by:

$$u_t(k_t, \underline{w}) = 1 - ((1-\alpha)A/\underline{w})^{\frac{1}{\alpha}} \cdot k_t, \tag{11}$$

which is positively related with the minimum wage and strictly decreasing in the per-capita stock of capital.

Once the wage has been fixed, the real rate of interest is exogenous (that is, capital returns are independent of the capital stock). A binding minimum wage, in fact, necessarily causes any increase of the capital stock to be matched by an identical increase of the employment level, keeping the capital-labour ratio constant over time. Indeed, substitution of (11) into (9) yields:

$$r(\underline{w}) = \alpha A((1-\alpha)A/\underline{w})^{\frac{1-\alpha}{\alpha}} - 1, \tag{12}$$

so that any increase of the minimum wage always pushes down the real interest rate below its competitive level.

In order to better clarify the meaning of the coefficient α (the capital's weight in technology), it is worth noting that a possible interpretation is that the capital stock may be thought in its broad concept, including physical and human components and that the labour input only includes non-specialised labour. In fact, as argued by Mankiw et al. (1992, p. 417), the minimum wage may be considered to be a proxy of the return to labour without human capital; they suggest that since the minimum wage has averaged about 30 to 50 percent of the average wage in manufacturing, then 50 to 70 percent of total labour income represents the return to human capital, so that if the physical capital's share of income is expected to be about 1/3, the human capital's share of income should be between 1/3 and one half. In sum, with the broad view of capital the coefficient α may be fairly about 0.6 and 0.8. Indeed, for instance, Barro and Sala-i-Martin (2003, p. 110) used $\alpha = 0.75$ saying that: "Values in the neighbourhood of 0.75 accord better with the empirical evidence, and these high values of α are reasonable if we take a broad view of capital to include human components".

2.5 Equilibrium. We now combine all the pieces of the model to characterise the long-run equilibrium. The market-clearing condition in goods as well as in capital markets is expressed by the equality between savings and investments, i.e. with the hypothesis of full depreciation of capital at the end of each period, and knowing also that $N_{t+1} = n_t N_t$ equilibrium implies $n_t k_{t+1} = s_t$, meaning that the stock of capital in period $t+1$ equals the amount saved in period t discounted by the number of individuals. Substituting out for n_t and s_t according to (6) and (7) and using eq. (11), the evolution of capital is driven by the following first order linear difference equation:

[14] We suppose that capital totally depreciates over time, i.e. $\delta = 1$. This assumption is not unrealistic in the present context, because as noticed by De La Croix and Michel (2002, p. 338) "even if one assumes a rather low annual depreciation rate of 5%, 79% of the stock of capital is depreciated after 30 years".

$$k_{t+1} = \mu\gamma\underline{w} + \mu(1-\gamma)((1-\alpha)A)^{\frac{1}{\alpha}}\underline{w}^{-\left(\frac{1-\alpha}{\alpha}\right)}k_t, \tag{13}$$

where $\mu := \phi q / \rho$. Steady-state implies $k_{t+1} = k_t = k^*$. Thus, the per-capita long-run stock of capital is given by:

$$k^*(\underline{w}) = \frac{\mu\gamma\underline{w}^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1-\alpha}{\alpha}} - \mu(1-\gamma)((1-\alpha)A)^{\frac{1}{\alpha}}}. \tag{14}$$

As regards stability, the solution of eq. (13) gives:

$$k_t = k_0 h^t + k^*(\underline{w}), \tag{15}$$

with $k_0 > 0$ given and $h := \mu(1-\gamma)((1-\alpha)A)^{\frac{1}{\alpha}} / \underline{w}^{\frac{1-\alpha}{\alpha}}$. Stability requires $h < 1$, that is $\underline{w} > w_T := (1-\gamma)^{\frac{\alpha}{1-\alpha}} \cdot w^*{}_c$, with $w^*{}_c := \mu^{\frac{\alpha}{1-\alpha}} \cdot ((1-\alpha)A)^{\frac{1}{1-\alpha}}$ being the steady-state market-clearing wage.[15] Since $w_T < w^*{}_c < \underline{w}$ holds true, the long-run equilibrium defined by (14) is globally stable whatever the minimum wage, i.e. $\lim_{t\to+\infty} k_t = 0^+$ for any $\underline{w} > w^*{}_c$.

Substituting out (14) into (11) and (8) gives the long-run unemployment rate and the long-run per-capita output level as a function of the minimum wage:[16]

$$u^*(\underline{w}) = \frac{\underline{w}^{\frac{1-\alpha}{\alpha}} - \mu((1-\alpha)A)^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1-\alpha}{\alpha}} - \mu(1-\gamma)((1-\alpha)A)^{\frac{1}{\alpha}}}, \tag{16}$$

$$y^*(\underline{w}) = \frac{A((1-\alpha)A)^{\frac{1-\alpha}{\alpha}}\mu\gamma\underline{w}}{\underline{w}^{\frac{1-\alpha}{\alpha}} - \mu(1-\gamma)((1-\alpha)A)^{\frac{1}{\alpha}}}. \tag{17}$$

Let $\alpha_k := 1-\gamma$ and $\alpha_y := 1/(1+\gamma)$ with $\alpha_y > \alpha_k$.[17] The following table summarises the parametric conditions under which the regulated-wage economy performs better than the competitive-wage economy as regards both the long-run capital accumulation and the long-run income.[18]

Table 1. *Critical values of the capital's weight in production (α), beyond which the regulated-wage economy performs better than the competitive-wage economy with respect to the per-capita long run capital accumulation and output.*

Condition for a higher capital accumulation	*Condition for a higher output*
$\alpha > \alpha_k := 1-\gamma$	$\alpha > \alpha_y := 1/(1+\gamma)$

Exploiting (14)-(17) together with the inequalities stated in Table 1, simple algebraic manipulations lead to the following remark:

[15] Finally, when the market-clearing wage prevails and thus the unemployment rate is zero, the standard steady state results of the Diamond OLG model in terms of capital per capita, output and fertility are respectively: $k^*{}_c = (\mu(1-\alpha)A)^{\frac{1}{1-\alpha}}$, $y^*{}_c = A(\mu(1-\alpha)A)^{\frac{\alpha}{1-\alpha}}$ and $n^*{}_c = \frac{\rho}{(1+\rho)q}$.

[16] Notice that, as regards the steady-state unemployment rate, the condition $0 < u^*(\underline{w}) < 1$ holds for any $\underline{w} > w^*{}_c$. To prove this result, it is sufficient to see that $u^*(\underline{w}) = 0$ if and only if the steady-state market-clearing wage prevails, i.e. $\underline{w} = w^*{}_c$ and $\lim_{\underline{w}\to+\infty} u^*(\underline{w}) = 1$.

[17] Note that $\alpha_y \to 1/2$ as γ approaches unity.

[18] Table 1 and Remark 1 straightforwardly derive from the analysis of the functions $k^*(\underline{w})$, $y^*(\underline{w})$ and $V^*(\underline{w})$. Details (which are available on request) are here omitted for economy of space.

Remark 1. *A necessary and sufficient condition for both the (transitional) rate of economic growth and the long-run income to be improved by the regulation of wages is* $\alpha > \alpha_y$.

Remark 1 says that, given the unemployment benefit policy, if the weight of capital in production is high enough, the regulated-wage economy is more efficient than the market-wage economy despite the unemployment occurrence.
Now, substituting out eq. (16) into eq. (6) to eliminate $u^*(\underline{w})$ we get:

$$n^*(\underline{w}) = \frac{\phi}{1+\rho}((1-\alpha)A)^{\frac{1}{\alpha}} \cdot \underline{w}^{\frac{\alpha-1}{\alpha}}, \tag{18}$$

Eq. (18) describes the relationship between fertility and minimum wage in the long-run. Using the latter equation, the following proposition holds:

Proposition 1. *The long-run fertility rate is always reduced by the regulation of wages and* $n^*(\underline{w}) < n^*{}_c$ *for any* $\underline{w} > w^*{}_c$.

Proof. *Differentiating (18) with respect to* $\underline{w}$ *we get* $\frac{\partial n^*(\underline{w})}{\partial \underline{w}} = -\frac{(1-\alpha)\phi}{\alpha(1+\rho)}((1-\alpha)A)^{\frac{1}{\alpha}} \underline{w}^{\frac{-1}{\alpha}} < 0$. *Since* $n^*(\underline{w}) = n^*{}_c$ *if and only if* $\underline{w} = w^*{}_c$, *it follows that* $n^*(\underline{w}) < n^*{}_c$ *for any* $\underline{w} > w_c$. ***Q.E.D.***

Proposition 1 stems directly from the role played by the unemployment rate: increasing the minimum wage always raises the long-run unemployment rate, and, given the negative correlation among fertility and unemployment, a higher wage ultimately implies a lower population growth. Interestingly, the latter proposition shows that the value of the regulated wage may also be treated as a policy parameter for controlling fertility choices.
It is important to note that the minimum wage legislation introduces a negative correlation among fertility and wages, whereas in the case of competitive labour market population is constant over time.

2.6 Welfare. After having discussed the economic growth and fertility outcomes of the model we turn to the welfare analysis, which has been carried out in terms of comparing steady state paths of the lifetime welfare of the representative generation, following, among many others, Samuelson (1975).[19]
A benevolent government is supposed to be a Stackelberg leader with respect to individuals and firms (Stackelberg followers). Given the followers' behaviour and knowing also that the lump-sum tax is an endogenous variable, the government chooses the minimum wage such as to maximise the steady-state indirect utility index of the representative generation, i.e.:

$$\max_{\{\underline{w}\}} V^*(\underline{w}) = (1-\phi)\ln(c^{*y}(\underline{w})) + \phi \ln(c^{*o}(\underline{w})) + \rho \ln(n^*(\underline{w})), \tag{19}$$

subject to (18) and

$$c^{*y}(\underline{w}) = \frac{1-\phi}{1+\rho}((1-\alpha)A)^{\frac{1}{\alpha}} \underline{w}^{\frac{\alpha-1}{\alpha}} k^*(\underline{w}),$$

$$c^{*o}(\underline{w}) = \frac{\phi}{1+\rho}(1+r(\underline{w}))((1-\alpha)A)^{\frac{1}{\alpha}} \underline{w}^{\frac{\alpha-1}{\alpha}} k^*(\underline{w}),$$

with $r(\underline{w})$ and $k^*(\underline{w})$ being determined by (12) and (14) respectively. We may then proceed to analyse the relationship among minimum wages and welfare. Differentiating (19) with respect to $\underline{w}$ yields:

[19] It is worth noting that in this paper we only perform a positive rather than a normative analysis.

$$\frac{\partial V^*(\underline{w})}{\partial \underline{w}} = \frac{1-\phi}{c^{*y}(\underline{w})}\frac{\partial c^{*y}(\underline{w})}{\partial \underline{w}} + \frac{\phi}{c^{*o}(\underline{w})}\frac{\partial c^{*o}(\underline{w})}{\partial \underline{w}} + \frac{\rho}{n^*(\underline{w})}\frac{\partial n^*(\underline{w})}{\partial \underline{w}}, \tag{20}$$

or

$$\frac{\partial V^*(\underline{w})}{\partial \underline{w}} := \Lambda(z) = \frac{[\alpha(2+\phi+\rho)-(1+\phi+\rho)]z - [\alpha(1+\phi+\rho)-(\phi+\rho)]z_T}{z^{\frac{\alpha}{1-\alpha}}\alpha(z-z_T)}, \tag{21}$$

where $z := \underline{w}^{\frac{1-\alpha}{\alpha}}$, $z_c := \mu \cdot ((1-\alpha)A)^{\frac{1}{\alpha}}$ and $z_T := (1-\gamma)z_c < z_c$. Let $\alpha_V := \frac{1+\gamma(\phi+\rho)}{1+\gamma(1+\phi+\rho)}$ with $\alpha_V > \alpha_y > 1/2$. The following remark holds:

Remark 2. *A necessary and sufficient condition for the lifetime welfare to be improved by the regulation of wages is* $\alpha > \alpha_V$.

Thus, provided a sufficiently high capital's weight in production, the regulation of wages brings always upon an increase of the welfare level than in the competitive-wage economy.
Moreover, differentiating α_V with respect to γ, ϕ and ρ we find that:

$$\frac{\partial \alpha_V}{\partial \gamma} = \frac{-1}{[1+\gamma(1+\phi+\rho)]^2} < 0, \tag{22}$$

$$\frac{\partial \alpha_V}{\partial \phi} = \frac{\partial \alpha_V}{\partial \rho} = \frac{\gamma^2}{[1+\gamma(1+\phi+\rho)]^2} > 0. \tag{23}$$

From eqs. (22) and (23), the following remark is straightforwardly derived:

Remark 3. *i) Any increase of the replacement ratio reduces the critical value of the capital's weight in technology for which the lifetime welfare is an increasing function of the regulated wage; ii) on the contrary, the more agents prefer to postpone consumption in future periods (*$\phi \uparrow$*), and the more parents are children-interested (*$\rho \uparrow$*), the higher the capitals' weight in technology needed to have an increasing welfare function.*

Of course, we note that in the short run the welfare effect of the introduction of minimum wages cannot be Pareto efficient (unless redistributive policies between the two generations living at the moment of the introduction of the regulation of wages, are implemented): indeed 1) the young generation may be better off (provided that the capital accumulation effect is sufficiently positive); 2) even if the welfare of the younger generation is increased, the old generation always incurs in a welfare loss due to the decreased interest rate. Anyway, the short run welfare analysis is beyond the scope of the present paper.
Therefore, given the unemployment benefit policy and provided a sufficiently high capital's weight in technology both the long-run economic growth and the lifetime welfare can be improved by the regulation of wages, despite the occurrence of unemployment. As a result, economic growth and welfare are positively linked with the unemployment rate.

3 A qualitative analysis

A simple numerical simulation, for a parametric configuration chosen only for illustrative purposes, may help us in evaluating how capital stock, income, welfare and fertility change along with the level of the regulated wage.
Figure 1 depicts the behaviour of the locus of accumulation of capital (see eq. (13)) showing that the regulation of wages implies, for the parametric configuration we have chosen, that the economy

moves along with a higher capital accumulation path than in the case of competitive labour market, resulting in a higher steady-state capital per-capita. This depends on the value of the capital's weight in technology, according to the conditions presented in Table 1.
Figures 2 and 3 show that long-run capital stock, income and welfare are increasing function of the regulated wage, indicating that the policymaker should fix as high a regulated wage as possible such as to obtain the possible highest long-run economic growth and welfare. Figure 4, instead, depicts the negative response of the fertility rate to increases in the regulated wage; due to this "modern" fertility behaviour, when the minimum wage is fixed at too high a level, population becomes stationary or it may even decrease. Overall the three figures clearly show that policymaker may choose a value of the regulated wage aiming to reach as high an output and welfare levels as possible compatible with the desired population growth rate.

[FIGURES 1-4 ABOUT HERE]

4 Conclusions

In this paper we have focused on the steady state effects of the regulation of wages on economic growth, welfare and fertility in a textbook OLG model of neoclassical growth.
Our results differ markedly from the conventional wisdom which argues that the minimum wage legislation always brings upon an efficiency loss. The reason for this wisdom is that it implicitly assumes a static context where production factors are fixed, whereas in a dynamic overlapping generations frame, where capital accumulation is affected by wages, such a wisdom might be incorrect. Indeed, in the paper we have showed that the regulation of wages could increase both economic growth and the lifetime welfare, on the one hand, and reduce the fertility rate on the other hand. Therefore, we conclude that under suitable conditions a regulated wage economy may perform better than a market-wage economy, and the level of the regulated wage may also be treated as a policy parameter for controlling population growth. Moreover we note that, particularly as regards underdeveloped as well as developing countries, where low wages, low economic growth, inadequate welfare states and high fertility rates are current stylised facts, our findings offer some interesting policy implications. The interest of these results lies in: 1) the relevance of their messages showing a new perspective for the regulation of wages, and 2) the simplicity with which are obtained, that is within a standard dynamic general equilibrium overlapping generations model where the departures from the textbook OLG frame are simply the assumptions of endogenous fertility and wages regulated by law.
In particular, in contrast with the prevailing literature, we have shown that introducing a labour market imperfection in a simple OLG setup may reverse the correlation between unemployment and economic growth, and the higher the long-run unemployment rate is the higher both the long-run output and the lifetime welfare will be.

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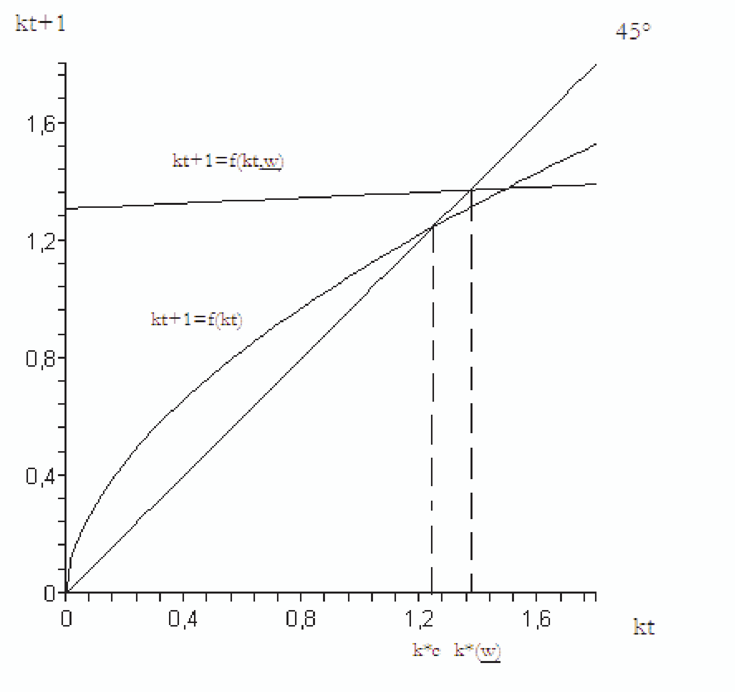

***Figure 1**. The capital accumulation path in the case of both the competitive-wage and the regulated-wage economies,* $k_{t+1} = f(k_t)$ *and* $k_{t+1} = f(k_t, \underline{w})$ *respectively. Parameter set:* $A = 100$, $\alpha = 0.56$, $\phi = 0.05$, $\gamma = 0.95$, $\rho = 0.10$ *and* $q = 0.05$.

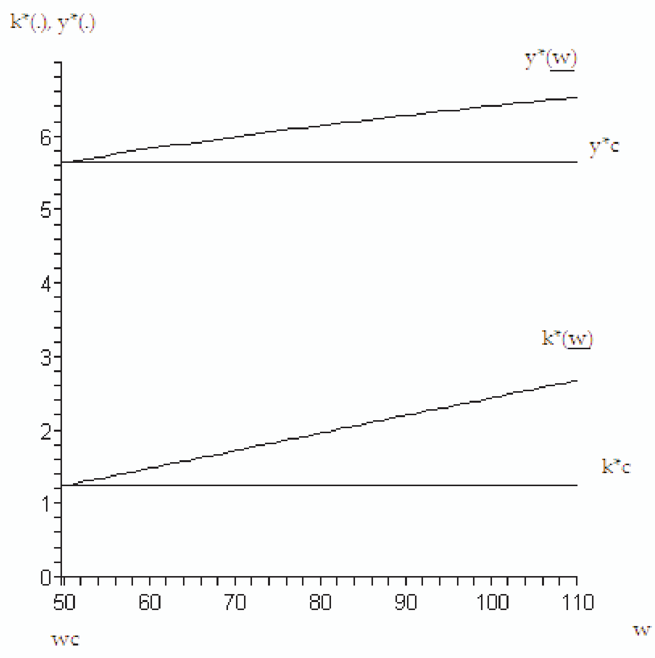

Figure 2. *The long-run stock of capital and the long-run income in both the market-wage (* w^*_c *) and regulated-wage (* $\underline{w}$ *) economies.* $y^*(\underline{w})$ *is scaled 1:10. The starting point of the horizontal axis is the market-clearing wage, that is* $w^*_c = 49.67$. *Parameter set:* $A = 100$, $\alpha = 0.56$, $\phi = 0.05$, $\gamma = 0.95$, $\rho = 0.10$ *and* $q = 0.05$.

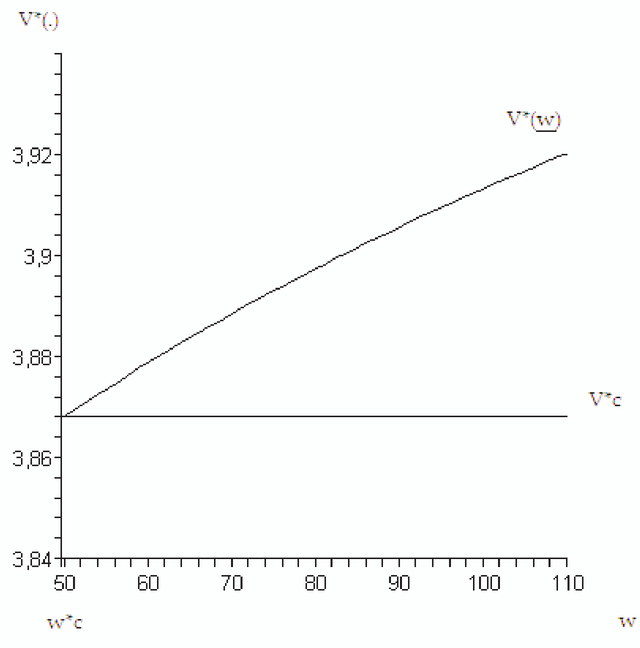

Figure 3. *The long-run lifetime welfare in both the market-wage (* w^*_c *) and regulated-wage (* $\underline{w}$ *) economies. The starting point of the horizontal axis is the market-clearing wage, that is* $w^*_c = 49.67$. *Parameter set:* $A = 100$, $\alpha = 0.56$, $\phi = 0.05$, $\gamma = 0.95$, $\rho = 0.10$ *and* $q = 0.05$.

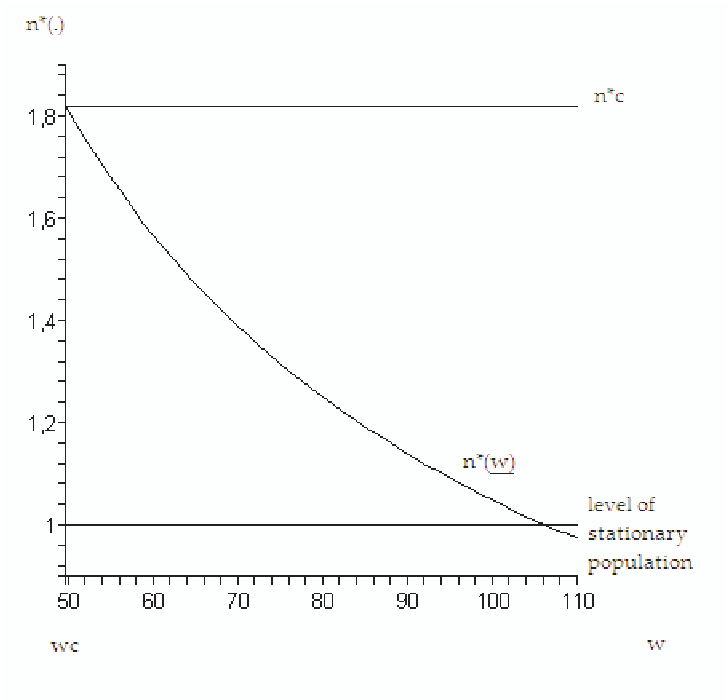

Figure 4. *The long-run fertility rate in both the market-wage (w^*_c) and regulated-wage ($\underline{w}$) economies. The starting point of the horizontal axis is the market-clearing wage, that is* $w_{pc} = 49.67$. *Parameter set:* $A = 100$, $\alpha = 0.56$, $\phi = 0.05$, $\gamma = 0.95$, $\rho = 0.10$ *and* $q = 0.05$.