Child support and custody arrangements in the bargaining family

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Preliminary Version

We use a two-period cooperative family bargaining model in which the (potential) parents choose the number of children, labor supply and consumption in the first stage. These choices determine the bargaining positions in the second stage which in turn has repercussions on the bargaining power of the partners. We consider a specific child support regime and custody arrangement and analyze its impact on the parents' decisions. We show that (and how) these legal arrangements matter for the bargaining power of the partners and that it does influence the number of children as well as labor supply and consumption in both periods and, hence, the couple's welfare.

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1 Introduction

This paper addresses the influence of child support and custody arrangements on the propensity to have children in a cooperative family bargaining context in which divorce is an outside option.

In nearly all the modern industrialized countries, divorces have got more and more likely during the past fifty years. The reasons for this development have been discussed already by Becker [Bec91][324 ff.] who relies on changes in the economic and social environment as explaining factors. More recent empirical studies provide detailed analyses of the driving forces of divorce (see Kesselring and Bremmer [KB06] for example).

Most divorces take place in families with children. In Germany, more than 50 % of all divorces affect at least one child under the age of 18[1]. It is straightforward that all Western countries have developed a more or less sophisticated legislation of child custody and child support payments. When potential parents think about having children, it is likely that they take the risk of divorce into account: If one of the parents knows in advance that it will be her duty to care for the children after divorce and child support payments will be low, it is likely that this will reduce her inclination to have children. It is also likely that changes in the legal environment concerning child custody and divorce will have an impact on this decision.

To model this connection, we use a two-period cooperative bargaining model with the two prospective parents as bargaining partners. The first period $t = 1$ is characterized by the early-stage partnership in which the decision about children takes place. Both partners have to decide whether they want to have children (and if yes, how many), how many hours they want to work in the market and how much they intend to consume in the first period. It is quite natural that, once the decision about children is made, it cannot be revised. It follows then that in the second period $t = 2$, the number of children is given. Once the number of children is fixed, their costs that occur to the parents are also known. So the parents decide only about labor supply

[1] Proksch [Pro02] provides an extensive statistical study on the distribution of custody arrangements and contacts after divorce between parents and children resp. between mother and father in Germany.

and consumption in the second period, taking their children's costs as given.
The salient feature of the bargaining model proposed here is that the parents' bargaining positions matter for the bargained outcome[2]. A strong outside option for a partner will improve his bargaining position in the partnership. That means that - if renegotiation is in principle possible - the couple determines its cooperatively bargained outcome *as well as* their individual bargaining positions in the second period by a decision in the first stage. As both are aware of these consequences, they will take them into account in the first period when deciding about the number of children.
The government (and the courts) may influence the situation of the divorced couple insofar as they create the legal basis for custody and child support arrangements. So the state may - via altering the legislation - indirectly influence the couple's propensity to have children and their welfare position.

To show this, the remaining paper is organized as follows: Section 2 presents and discusses the whole two-period model. As this will be solved recursively, section 3 solves for the second period, treating the number of children as exogenous variable. Section 4 presents the setting of the first period in which the influence of the child custody legislation on the initial decision about children will be shown. Section 5 concludes.

2 The model

We consider two partners $i = m, f$ with utility functions $U_i^t = f(x_i^t, l_i^t, y)$ that are defined over consumption x, labor supply l and children y in period t. The function U_i is concave in consumption and children resp. convex in labor supply (this corresponds to concavity in leisure). The consumption good's price is set equal to 1 while the wage rates for both partners are w_i. The variable y is a continuous variable. It can be understood as quality-weighted number of children which captures the quantity as

[2]Family bargaining models of this type have been discussed by many authors during the past 25 years, see for instance Manser and Brown [MB80], McElroy and Horney [MH81], Lunderg and Pollak [LP96], Lundberg et al. [LSS03], Pollak [Pol05] and Gugl and Welling [GW06].

well as the quality of the children[3]. The costs of children are assumed to be expressed in a cost function $c(y)$ which has the usual properties[4].
The government raises an income tax with a constant marginal tax rate T. Further refinements could be introduced here like a lump-sum transfer, a direct progressive tax schedule or different marginal tax rates for the partners. But as the main focus of this work is on custody arrangements and child support legislation this will be sufficient.

The threat points are defined as the situation of the partners as singles. This is the fall-back utility that they can achieve on their own if the partnership ends. We will denote the utility as a single person of partner i in period t as V_i^t. In the first period the threat point is naturally determined by the situation as a single person without children: Both partners decide either to stay alone or to begin the partnership (see the structure of the bargaining game in appendix C).
In the second period the specification of the threat point depends on the liability of the partnership. We assume that both partners have the possibility to renegotiate after the first period, i.e. when the decision about the number of children is made. It then follows that the threat point in the second period (V_i^2) is the utility that each spouse would achieve in the case of divorce. For V_i^2 it is crucial who gets the custody of the children in case of divorce and who has to support the children by payments. This will be shown in section 3.

We can describe the whole game as

$$\max \sum_t \sum_i (U_i^t - V_i^t) \tag{1}$$

Both partners maximize the sum of their Nash dividends. i.e. their utility gains from the partnership. As mentioned earlier, the number of children y is the only variable that is determined exclusively in the first period. In the second period, y is fixed and so are the costs of the children. This influences the budget in the bargaining game of the second period *as well as* the budget constraints of the partners in case of divorce. Hence, the utilities of the partners in the marriage in the second period $U_i^2(\overline{y})$ as well

[3]See Cigno and Pettini [CP02] for a similar treatment. They assume that child quality is a concave function of attention devoted to the child and commodities.

[4]See Mas-Colell et al. [MCWG95][135 ff.] for example.

as the utilities as singles $V_i^2(\bar{y})$ depend on the decision about y which is made at the beginning of the first period and which cannot be reversed.

The absence of binding agreements raises the well-known problem that we may have an intertemporally consistent equilibrium which is not Pareto efficient[5]. Intertemporal concistency is closely linked to the concept of subgame perfectness. Güth and Selten [GS82] applied the concept of subgame perfectness for multi-period cooperative bargaining in which the threat points in subsequent periods depend on decisions in former periods. We will also assume subgame perfectness, i.e. we will assume that we get optimal solutions for the whole bargaining game (which is played at the beginning of period 1 and period 2) as well as the subgame which is played only in period 2.

We will solve the bargaining game by backward induction. Firstly, the results of utility maximization for the case that the partners get divoced at the beginning of period 2 will be presented. This determines V_i^2. Secondly, we will use them to derive the results of the optimization process in period 2. This yields U_i^2. The bargained utility U_i^2 as well as the threat points V_i^2 will both be used to solve the bargaining problem of period 1 which is identical to eq. (1).

3 The second period

3.1 The divorce case

Related literature

If the partnership fails and both partners opt for a divorce they have to decide how to allocate the child custody and the support settlement concerning the children[6]. There is a wide range of literature that has emerged around the issue of child support payments and legal custody which can broadly be divided into two strands. One takes child support payments as dependent, one takes it as independent variable.

[5] See Jorgensen and Zaccour [JrZ02] and Sorger [Sor06] for a discussion.

[6] We will omit the presence of support payments to the former partner. Nevertheless this could also be integrated into the budget constraint.

Especially Weiss and Willis ([WW85] and [WW93]) have significantly contributed to explain child support payments after divorce. In their model (and also in most of the subsequent post-divorce models including the one presented here) children are a collective good within the household. As soon as the household dissolves and the parents get divorced, it cannot be consumed to the same extent by both parents. Furthermore, they assume that the father (who has to support the children) cannot control the use of his resources because they are entirely at the mother's disposition. If she uses the payments for own consumption purposes, the child (and, hence, the father) does not benefit. Willis and Weiss argue that this is the main reason for the fact that most fathers reduce payments after divorce. As another reason they state the unfavorable treatment of the received payments by tax and social law in many contries (like the UK) with imlicit tax rates up to 100 %.
Recently, other determinants of child support have been proposed. If child support payments are taken together with time spent with the children, these investments seem to correlate with the paternal certainty. Anderson et al. [AKL07] show for a sample of 1325 men in Albuquerque that there is a close link between paternal investments in children living away from him and his subjective assessment to be the biological father. Ermisch and Pronzato [EP06] argue that fathers decrease child support payments whenever they have a new partnership because they feel the duty to redirect resources to their "new family". They test this hypothesis using the British Household Panel Survey.

Maintainance payments are also used as independent variable. If the mother has sole custody and the father is more or less free to make payments or not (because the legal enforcement is not strong enough) he can use the payments to buy time to spend together with his children. If he makes generous payments, the mother may be inclined to let him spend more time with his children. This line of reasoning can be found in Del Boca and Ribero [DBR03] and Ermisch [Erm05].
If in the contrary support payments are enforced by the courts and fathers are indeed obliged to pay, this may affect the inclination of both partners to divorce. Zhu and Walker [WZ06] use data from the United Kingdom to show this correlation. Huang [Hua05] even claims that tight legislation concerning child support payments

reduces the probability of unintended pregnancies but his analysis is restricted to US data (National Longitudinal survey of Youth); it is not clear whether this also applies for European countries.
A rather surprising effect of support payments could be that those resources improve the chances of remarriage for the former spouse on the remarriage market (see Chiappori and Weiss [CW07]). This in turn reduces the expected future payments of the former husband so that it could be worth raising his payments above the level that he would opt for normally.
But none of the above cited articles treats fertility decisions as endogenous (except for Zhu and Walker and Huang which are both empirical treatments without a decision model) as it is done in this framework.

Specification of threat points

We assume that m is the partner who has to make payments to f with who the children live after divorce. These payments are determined by legislation and dependent on his labor income: He has to pay a share of $\tau \in [0;1]$ of his labor income per child.
The mother f in turn has to carry the costs for the children $c(y)$. Though the children live mainly with her, m has the right to spend time with them. We assume that the government resp. the courts also set this amount of time and capture this by the notation that the children spend a $\theta \in [0;1]$ share of their time with their father and the remaining time $1-\theta$ with their mother. It is likely that both parents will only benefit from their children when they are with them so that the maximization problem of f can be written as[7]

$$\max \quad U(x_f, l_f, (1-\theta)y) \text{ s.t. } (1-T)w_f l_f + \tau w_m l_m y - c(y) = x_f. \tag{2}$$

The problem of m then is

$$\max U(x_m, l_m, \theta y) \text{ s.t. } (1-T-\tau y)w_m l_m = x_m. \tag{3}$$

As mentioned above, y, w_i, τ, θ and T are exogenous resp. treated as exogenous as in the case of y.

[7]Note that all time indices t are omitted in this section because it is clear that all variables refer to the second period. Nevertheless we will use indices in section 4 as both periods are analyzed.

We get as a first-order condition for f and m concerning optimal labor supply[8]:

$$\frac{\frac{\partial U_f}{\partial l_f}}{\frac{\partial U_f}{\partial x_f}} = -(1-T)w_f \quad \text{resp.} \quad \frac{\frac{\partial U_m}{\partial l_m}}{\frac{\partial U_m}{\partial x_m}} = -(1-T-\tau y)w_m \tag{4}$$

We can easily see the difference in the right-hand sides of the equations: While the marginal rate of substitution of f must equal her wage rate minus tax, the appropriate price of one unit of labor for m is his wage rate minus tax *minus* child support payment per unit of labor (τy). Hence, the child support payments act as an additional tax on his labor income and will reduce his labor supply compared to a similar situation without payments to be made.

The following is dedicated to comparative static analysis. As we need to know how the threat points change when exogenous parameters change for solving the whole problem in period 1, we will present the results of an increase in the number of children first. Afterwards a change in τ is considered as this is the most interesting parameter for government intervention. For the sake of simplicity we will assume quasilinear preferences of the type $U(x_i, l_i, y) = x_i + f_1(l_i) f_2(y)$[9]. This is a restriction which is nevertheless often used in articles on optimal income taxation (see Diamond [Dia98] or Hamilton and Pestieau [HP05] for example). Konrad and Lommerud assume quasilinear preferences in a household production context [KL00].
Labor supply of m will react in the following manner:

$$\frac{dl_m}{dy} = \frac{\tau w_m - \frac{\partial^2 U_m}{\partial l_m \partial y}}{\frac{\partial^2 U_m}{\partial l_m^2}} \tag{5}$$

As this is negative, m decreases his labor supply when divorced in the second period. As (6) shows he will also decrease his consumption:

$$\frac{dx_m}{dy} = (1-T-\tau y)w_m \frac{dl_m}{dy} - \tau w_m l_m \tag{6}$$

[8]The complete first-order conditions as well as the comparative static analysis for all problems presented here can be obtained from the author. As they are standard techniques they are not displayed in this paper.

[9]Note that this implies $\frac{\partial^2 U}{\partial l \partial y} = \frac{\partial^2 U}{\partial y \partial l} < 0$

As m's decrease in working hours leads to decreasing incomes this result is not surprising.
Concerning f the results are slightly different:

$$\frac{dl_f}{dy} = -\frac{\frac{\partial^2 U_f}{\partial l_f \partial y}}{\frac{\partial^2 U_f}{\partial l_f^2}} \tag{7}$$

She will also decrease labor supply but the reaction of consumption is not clear:

$$\frac{dx_f}{dy} = (1-T)w_f \frac{dl_f}{dy} - \frac{\partial c}{\partial y} + \tau y w_m \frac{dl_m}{dy} + \tau w_m l_m \tag{8}$$

On the one hand her own labor income decreases because f works less. On the other hand she gets more payments from m as there are more children to maintain. Hence, if the labor income of m is comparably high, it could occur that f increases her consumption despite she works less.

Now we are interested in the changes in the threat positions of the partners because of an increase in τ, i.e. an increase in the share of the child support payments per child. We get

$$\frac{dl_m}{d\tau} = \frac{y w_m}{\frac{\partial^2 U_m}{\partial l_m^2}} \tag{9}$$

for m's reaction of labor supply, which will decrease, as well as

$$\frac{dx_m}{d\tau} = (1-T-\tau y) w_m \frac{dl_m}{d\tau} - y w_m l_m \tag{10}$$

which shows that he will decrease consumption which is partly due to his decreased labor supply (first term on rhs) and partly due to his increased support payments (second term on rhs).

For the subsequent analysis of the first period, we still need the changes in the threat points as reactions to a change in the number of children y. Using (5) and (6) we get

$$\frac{dV_m}{dy} = \frac{\partial U_m}{\partial y}\theta - \tau w_m l_m \tag{11}$$

as the change in the bargaining position for m if the number if children changes. Expression (11) is a very neat demonstration of the two opposite effects of having an additional child for m: The left-hand expression on the right-hand side shows the changed utility arising from a change in y (multiplied by θ as m has to share it with f in case of divorce) while the right-hand expression refers to the changed amount of child support. If m and f decide to have more children in the first period, m would benefit from more utility from this children (except for the case that he does not see the children at all in case of divorce ($\theta = 0$)) but he would also have to contribute more to their costs.

Analogously, a change in the reference position for f can be derived using (7) and (8):

$$\frac{dV_f}{dy} = (1-\theta)\frac{\partial U_f}{\partial y} - \frac{\partial c(y)}{\partial y} + \tau y w_m \frac{dl_m}{dy} + \tau w_m l_m \qquad (12)$$

The first two expressions on the right-hand side are an analogon to equation (11): On the one hand, f would benefit from more children as they bring more joy, on the other hand f is confronted with risen costs in the case of divorce. But the main difference that can be seen is that f *additionally* benefits from the change in support payments received from m. The first expression ($\tau y w_m \frac{dl_m}{dy}$) represents the decrease in the payments caused by the reduced labor supply of m while the second ($\tau w_m l_m$) refers to the increased payment due to the increased number of children. As this changes f's consumption opportunities it also affects her threat utility.

For the analysis in period 1 we also need some further differentiation of $\frac{dV_i^2}{dy}$ with respect to y and τ. The parameter τ will be analyzed in detail in the comparative static analysis. As this is just another comparative static exercise it can be found in the appendix A.

This subsection was intended to show the situation of both partners in the case of divorce. Hence, we have determined the threat points V_i^2 as well as their reactions to changes in the number of children y. Since the spouses know at the beginning of their partnership that it may end at the beginning of the second period, it is quite natural that they take their respective bargaining positions in that period into account; if one of the partner knows in advance that he or she will not benefit from (additional) children in case of divorce and that a divorce is likely, he or she will be reluctant to consent to having a child. It will be shown in section 4 that the bargaining position influences the couple's wish to have children.

3.2 The bargaining game in period 2

For the bargaining couple of the second period, the number of children is also fixed as it was decided in the first period. The only thing that has to be determined is labor supply and consumption of both partners.

In this subgame we assume a slightly different version of the bargaining rule applied for both periods. Both partners bargain in a way that maximizes the sum of their reference position and the splitted surplus from the partnership. We assume that the partners apply a splitting rule $e \in [0;1]$ which determines the share of m of the surplus $\sum_i U_i - V_i$. Consequently, f will get $1-e$. So both partner maximize their respective utilities, determining their labor supply and consumption in the second period. Afterwards their joint surplus is splitted between them according to the splitting rule[10]. Furthermore both partners split their income according to a splitting rule $\beta \in [0;1]$. An alternative setting would be to let any partner use only his own labor income. We found it more convincing to assume that both pool their labor incomes (because it may be likely that one spouse does part-time work or does not work at all when having children) and split it. The cost of children $c(y)$ are also split between the partners. Here it is likely that each spouse will carry the same amount so that an equal split applies[11].

Spouse m then maximizes

$$eU_m + (1-e)V_m - eV_f + \lambda_m(\beta(1-t)\sum_i w_i l_i - x_m - 0.5c(y)) \tag{13}$$

and f will maximize

$$(1-e)U_f - (1-e)V_m + eV_f + \lambda_f((1-\beta)(1-t)\sum_i w_i l_i - x_f - 0.5c(y)). \tag{14}$$

for f.

The comparative static analysis reveals differences between the situation in case of divorce and the "harmonic" case in which the partnership is continued in the second period. In the latter situation there are no asymmetric shocks like a change in τ from which only one partner benefits in the case of divorce. A change in the parameters

[10] Gugl [Gug06] has applied a similar model but restricted the splitting rule to be 0.5 ("equal split").

[11] Of course any other splitting rule would also be possible and would not change the results.

might affect one partner in particular in the first round (like a rise of one wage rate) but as cooperation is continued and the surplus is divided between the partners, both will benefit in the end.

Again, we look at how the spouses' situation is affected by a change in the number of children in the first period. Suppose that they decide to have more children. Then labor supply of both spouses decreases:

$$\frac{dl_i}{dy} = -\frac{\frac{\partial^2 U_i}{\partial l_i \partial y}}{\frac{\partial^2 U_i}{\partial l_i^2}} \tag{15}$$

The reason for this is simply the increasing marginal disutility from labor supply.

It is clear that more children mean more costs in the second period. Furthermore, we know from (15) that both spouses decrease labor supply so that both have less consumption possibilities:

$$\begin{aligned} \frac{dx_m}{dy} &= \beta(1-T)[w_m\frac{dl_m}{dy} + w_f\frac{dl_f}{dy}] - 0.5\frac{\partial c(y)}{\partial y} \\ \frac{dx_f}{dy} &= (1-\beta)(1-T)[w_m\frac{dl_m}{dy} + w_f\frac{dl_f}{dy}] - 0.5\frac{\partial c(y)}{\partial y} \end{aligned} \tag{16}$$

Both consumption responses will be identical if the labor income will be splitted equally between the spouses ($\beta = 0.5$). In equation (16), we see exactly the two effects at work: The first factor refers to the declining labor income of both spouses, the second one refers to the increased cost of the children.

Finally, we need to establish the change in utility in period 2 when the number of children in period 1 increased. Taking derivatives of U_i and plugging (15) and (16) into the expression yields (for m):

$$\frac{dU_m}{dy} = \frac{\partial U_m}{\partial y} - 0.5\frac{\partial c(y)}{\partial y} + \beta(1-T)w_f\frac{dl_f}{dy} \tag{17}$$

The first two effects are quite obvious: because of the positive marginal utility of children, the utility in period 2 will increase when the couple has more children. Nevertheless, more children decrease the consumption possibilities of the parents which will decrease utility in period 2. The third expression is interesting as it refers to f. This can be explained by the fact that f reduces labor supply because there are more children. Hence, the labor income of both (and consequently, also m's share) is decreasing. Spouse m also decreases labor supply but the associated loss of consumption opportunities is compensated by the increase in utility because m works less. But m does not benefit from the utility gain associated with the decrease of f's working hours.

The expression for $\frac{dU_f}{dy}$ can be derived analogously. It can be found in the appendix B as well as the further differentiations that are needed to solve the problem of the first period.

We have now traced the conditions of a solution of the second period bargaining problem. It is characterized by a decentralized bargaining problem that can be solved properly. Each spouse maximizes her fall-back utility (which in turn was characterized in subsection 3.1) plus half of the Nash dividend which can be achieved in the partnership. The solution of the subgame in period 2 assures the subgame perfectness of the total game, beginning in period 1.

4 The first period

Now the total game (1) will be solved in which we will use the results of the two preceding subsections. The two partners decide on consumption and labor supply (as in the second period) *but also* on the number of children they want. The number of children, as we put it earlier, has implications not only for the first period but also for the second period where y cannot be influenced anymore. This was shown in section 3. Keeping this in mind, both spouses are aware that their choice of the number of children has repercussions on their life in the second period if they stay together but also if not. Both know that in case of divorce there will only be one who lives with the children and one who will have to make support payments. As they know this in advance, they can integrate this in their reflections on how many children they would like to have. A potential disadvantage in a future bargaining position can easily be compensated through their intrafamily bargaining mechanism.

As already set up in section 2, the bargaining process can be written as the maximization of

$$\max_{x_i^1, l_i^1, y, \lambda} \sum_t \sum_i (U_i^t - V_i^t) + \lambda((1-T)\sum_i w_i l_i^1 - \sum_i x_i^1 - c(y)) \tag{18}$$

We will skip the optimality conditions referring to optimal labor supply and consumption because they do not deviate much from standard theory.
The intertemporal optimality condition for having children instead is more interesting:

$$\sum_i \frac{\partial U_i^1}{\partial y} + \sum_i \frac{dU_i^2}{dy} - \sum_i \frac{dV_i^2}{dy} = \frac{\partial c(y)}{\partial y} \tag{19}$$

Equation (19) shows the logic of the whole bargaining process: Not only the costs (right-hand side) and benefits (first term left-hand side) of the *first* period are important but also those of the *second* period. The latter is represented by the second and the third term on the left-hand side. The second term is identical with (17) while the third term has been analyzed in section 3.1 (eqs. (11) and (12)).

The term which represents the change in the bargaining position of the second period is particularly interesting. The bargaining position (= the maximum utility in case of divorce) appears in this setting only because renegotiation is possible in the second period. We could - in principle - also imagine binding contracts which means that the spouses are able to enforce their first-period agreements for the rest of their lives. A possible divorce (and, hence, any custodial regulations and payment schemes) would be of no interest for their bargaining mechanism. Consequently, the bargaining positions of the second period, V_i^2, would not appear in the marginal conditions and would also not influence the inclination to have children.
But it is clear that in most industrialized societies divorce is possible. It then follows that the potential parents which begin to think about their number of children are confronted with a certain probability that their partnership fails. We think that it is very unlikely that this - undesired, but not impossible - situation does not enter into the considerations of partners that want to commit themselves to each other for a long time.

It is consequently within the logic of this model that the government legislation has an impact on the decision on children. A rise in τ will be considered. Although this policy measure influences - in a first stage - only the partners' situation when divorced it alters their relative welfare position: m has to pay more and f receives it, so one could argue that it constitutes just a redistribution. But as shown in section 3.1, this leads to asymmetric changes in the bargaining positions of the couple. And as they anticipate this from the beginning, this has also repercussions on the couple's propensity to have children. Although this seems complex, it is realistic: The decision to have children has many long-term implications and usually this decision is made carefully.

As period 1 *and* period 2 are concerned by a rise of τ and several repercussions have to be considered, it is clear that the expression showing the change in the number of children is quite complex:

$$\frac{dy}{d\tau} = \frac{\sum_i \frac{d^2 V_i^2}{dy d\tau}}{\sum_i \frac{\partial^2 U_i^1}{\partial y \partial l_i} dl_i^1 + \sum_i \frac{\partial^2 U_i^1}{\partial y^2} - \frac{\partial^2 c(y)}{\partial y^2} + \sum_i \frac{d^2 U_i^2}{dy^2} - \sum_i \frac{d^2 V_i^2}{dy^2}} \tag{20}$$

Note that $dl_i^1 = -\frac{\frac{\partial^2 U_i^1}{\partial l_i \partial y}}{\frac{\partial^2 U_i^1}{\partial l_i^2}} < 0$. All the expressions in the numerator and the denominator concerning the change in utility resp. the threat point in period 2 have been treated in section 3 and in the appendix.

The numerator term can be written as

$$\sum_i \frac{d^2 V_i^2}{dy d\tau} = \underbrace{\theta \frac{\partial^2 U_m^2}{\partial y \partial l_m} \frac{dl_m}{d\tau}}_{>0} \underbrace{+ y w_m \frac{dl_m}{dy}}_{<0} \tag{21}$$

It can be seen that the sum of the changes in the marginal utility of children of both partners depends only on m's preferences. This is a simple demonstration of the fact that $d\tau$ leads to an asymmetric change. Though partner f who has the custody is also affected by a change in the payment this will be canceled out by the increased payments on m's side. But m's utility change connected with the decreased labor supply has no correspondent term in f's utility maximization problem.

The expressions in the denominator can be grouped: The first three expressions show the change in marginal benefits and costs that occur to the couple in the first period. They consist of a decrease in marginal utility (second term), an increase in marginal costs (third term) and a change in marginal utility of children due to a change in labor supply.

The fourth expression shows the change in marginal utility in period 2. It contains three expressions that are logically equivalent to the three first terms in the denominator. Hence, its interpretation is analogous.

Equivalently, the last expression shows the change in the marginal utility in the divorce situation when y changes. This term is a bit different:

$$\sum_i \frac{d^2V_i^2}{dy^2} = \frac{\partial^2 c(y)}{\partial y^2} - \theta^2 \frac{\partial^2 U_m^2}{\partial y^2} - (1-\theta)^2 \frac{\partial^2 U_f^2}{\partial y^2} \\ - (\tau w_m + \theta \frac{\partial^2 U_m^2}{\partial y \partial l_m^2}) \frac{d\tilde{l}_m}{dy} - (1-\theta) \frac{\partial^2 U_f^2}{\partial y \partial l_f^2} \frac{d\tilde{l}_f^2}{dy} \tag{22}$$

We see that the first three terms again count for the change in benefits and costs of children. However, they are weighted with the time share in which the children live with the respective partner. Another asymmetry arises from the fact that the (square) weights do not sum up to 1.

The expressions in the second line refer to the changes in marginal utility of children when labor supply is changed. Interestingly, we see again that the effect of $d\tau$ is different for f and m: While f changes her labor supply only because the number of children changes, m will change it because of both, the increase in τ and the subsequent change in the number of children.

We will not display the resulting changes in x_i^1 and U_i^1 because that was done in detail for period 2 in the preceding sections. It is nevertheless straightforward that the asymmetric changes in child support or custody leads to changes in the bargained utilities and the aggregate surplus $\sum_i (U_i - V_i)$ in the first period.

To sum up, it could be shown that (and how) the child support provisions matter in a two-period cooperative bargaining model. Even though they apply only in case of divorce and even though they are relevant only for the second period, the potential parents will take these provisions into account when deciding about their desired number of children.

5 Conclusion

This paper looks at a theoretical explanation of the influence of child support and custody on the propensity to have children. We propose a cooperative two-period bargaining model in which two spouses determine their number of children, labor supply and consumption in the first period. In the second period the couple faces a probability of divorce in which case they have to find a custody and child support payment scheme.

In a first step, we established the reactions of labor supply, consumption and welfare of both partners to changes in the number of children as well as to a rise in the child support parameter τ. These have well-defined signs and can be interpreted properly. In a second step we solved for the bargaining problem of the second period. Treating the number of children as exogenous variable, we also demonstrated how a change in the number of children influenced labor supply and consumption of the couple.

The results of the first two steps were used to solve the bargaining problem in the first period. The conditions which determine the optimal choice of the couple were presented and analyzed. We were able to show that and how governmental policies concerning the child support payments influence the couple's decision on children.

In our model, it is the government (resp. the court) who allocates the custodial rights between the partners via the parameter θ. It furthermore determines the amount of child support as a share τ of the labor income of the non-cohabiting spouse. As our framework is quite general it is natural to think about other possible child support and custody schemes. As one example, we could think about a setting similar to Ermisch's model [Erm05] in which the mother has sole custody. The higher the support payment is the more often she is willing to let the father visit his children. In terms of our notation, we would then endogenize θ. Another variant could be that the time allocation of the children is set by goverment but father decides how much he wants to contribute.

There are many other possible legal schemes and it would not be difficult to integrate them into the setting of the divorced couple. They would determine the couple's bargaining position V_i^2 and, hence, the decision on children.

Appendix

A Differentiation of the threat points

We take the threat point of m in the second period, V_m^2, as example. The derivation for f's threat points is completely analogous.
Differentiating (11) with respect to τ yields

$$\frac{d^2V_m}{dy\,d\tau} = \underbrace{(\frac{\partial^2 U_m}{\partial y \partial l_m}\theta - \tau w_m)\frac{dl_m}{d\tau}}_{>0} \underbrace{-w_m l_m}_{<0}. \tag{23}$$

Again, we see both effects of a change in y (caused by $d\tau$): A change in (marginal) utility of y as well as a change of consumption possibilities because of changed support payment patterns.
Differentiation with respect to y yields

$$\frac{d^2V_m^2}{dy^2} = \theta^2\frac{\partial^2 U_m}{\partial y^2} + (\frac{\partial^2 U_m}{\partial y \partial l_m}\theta - \tau w_m)\frac{dl_m}{dy} \tag{24}$$

A similar procedure can be applied to expression (12) and with respect to any other exogenous variable.

B Differentials of the second period utility

The change of f's utility due to changes in the number of chilren can be written as follows:

$$\frac{dU_f}{dy} = \frac{\partial U_f}{\partial y} - 0.5\frac{\partial c(y)}{\partial y} + (1-\beta)(1-T)\frac{dl_m}{dy} \tag{25}$$

This can easily be interpreted in the same way as (17).

For the comparative static analysis we also need the second derivative of the second period utility with respect to the number of children:

$$\frac{d^2U_i}{dy^2} = \underbrace{\frac{\partial^2 U_i}{\partial y^2}}_{<0} \underbrace{+\frac{\partial^2 U_i}{\partial y \partial l_i}\frac{dl_i}{dy}}_{>0} \underbrace{-0.5\frac{\partial^2 c(y)}{\partial y^2}}_{<0} \tag{26}$$

Again, it can be seen that this expression is well-defined and shows the derivation of the marginal effects of having children: The direct marginal utility of children decreases while the marginal utility of children increases with decreasing labor supply. Marginal cost also increases.

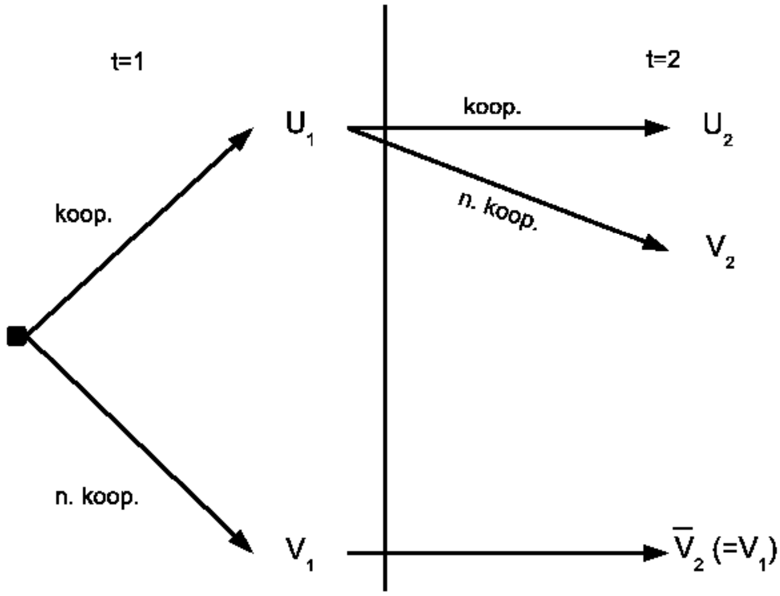

Figure 1: The structure of the game (non-binding agreements)

C Structure of the bargaining game

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