A Prize to Give for: An Experiment on Public Good Funding Mechanisms*

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March 2007

Abstract

This paper investigates fund-raising mechanisms based on a prize as a way to overcome free riding in the private provision of public goods, under the assumptions of income heterogeneity and incomplete information about income levels. We compare experimentally the performance of a lottery, an all-pay auction and a benchmark voluntary contribution mechanism. We find that prize-based mechanisms perform better than voluntary contribution in terms of public good provision after accounting for the cost of the prize. Comparing the prize-based mechanisms, total contributions are significantly higher in the lottery than in the all-pay auction. Focusing on individual income types, the lottery outperforms voluntary contributions and the all-pay auction throughout the income distribution.

Keywords: Auctions; Lotteries; Public Goods; Laboratory Experiments
JEL codes: C91; D44; H41

*We would like to thank Roberto Reale for his contribution to this paper. We also thank Ed Hopkins for useful comments and suggestions. We gratefully acknowledge financial support from Econometica.

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1 Introduction

Finding effective fund-raising mechanisms for the private provision of public goods is an important policy question. Voluntary contributions to public goods are typically well below socially optimal levels, given the incentive to free ride associated with positive externalities.[1] While fund-raising mechanisms based on tax rewards and penalties can be designed to overcome the incentive to free ride, they are not available to fund-raisers in the private sector who cannot enforce sanctions. A number of recent studies have examined, both theoretically and empirically, the performance of incentive-based funding mechanisms for the private provision of public goods, focusing in particular on lotteries (or raffles) and different types of auctions (e.g. Morgan, 2000; Morgan and Sefton, 2000; Goeree et al., 2005; Orzen, 2005; Schram and Onderstal, 2006).[2]

In this paper we investigate with a laboratory experiment the performance of prize-based mechanisms for the private provision of public goods, under the assumptions of income heterogeneity and incomplete information about income levels. We focus on a voluntary contribution mechanism, used as a benchmark, and two incentive-based mechanisms where a single prize is awarded: a lottery and an all-pay auction.

The experimental literature on incentive-based fund-raising mechanisms has focused on the case of income homogeneity (e.g. Morgan and Sefton, 2000; Orzen, 2005; Schram and Onderstal, 2006). However, actual contribution to public goods is generally characterised by heterogeneous incomes which are private information. Although several experimental studies have investigated public good provision when incomes are heterogeneous, this literature has only explored the voluntary contribution mechanism.[3] The performance of incentive-based fund-raising mechanisms when subjects have different incomes remains empirically unexplored.

Morgan (2000) provides a theoretical analysis of lotteries as a way to finance public goods. Players buy tickets of a lottery in which one prize is awarded. One ticket is randomly drawn and the holder wins the prize.

[1]Voluntary contributions to public goods are generally found to be greater than theoretical predictions, both in naturally occurring situations and in laboratory experiments, but nevertheless sub-optimal. See Ledyard (1995) for a survey of the experimental literature on the provision of public goods. See also e.g. Keser (1996), Laurie and Holt (1998), and Saijo (2003) for alternative explanations of over-contribution in the voluntary provision of public goods.

[2]See also e.g. Isaac and Walker (1988), Bagnoli, and Lipman (1989), Bagnoli and McKee (1991) for earlier studies on mechanisms for improving economic efficiency in the voluntary provision of public goods.

[3]Research has examined the effects of income heterogeneity on either overall public good provision (Anderson et al., 2004; Chan et al., 1996; Chan et al., 1999; Rapoport and Suleiman, 1993) or contributions of individual income types (Buckley and Croson, 2005).

Public good provision consists of the revenue of the lottery net of the prize. The author considers agents with heterogeneous preferences and endowments who have quasi-linear utility functions. Public good provision is shown to be strictly higher than with voluntary contributions. The solution identified by Morgan (2000) predicts that agents with different incomes contribute the same amount in equilibrium. Such an equilibrium does not seem realistic, while it appears more plausible that the contribution would be increasing in the endowment. Morgan and Sefton (2000) investigate experimentally the performance of a linear version of Morgan's model, finding that, as predicted, public good provision via a lottery is higher than through voluntary contributions. However they only consider homogeneous endowments, without testing the validity of a completely symmetric equilibrium.

Orzen (2005) compares in a laboratory experiment the performance of a lottery and different all-pay auctions as fund-raising mechanisms, under the assumptions of homogeneous preferences and endowments. Public good provision generated by the incentive-based mechanisms is higher than voluntary contributions. Interestingly, although theory predicts that the first price all-pay auction raises a higher revenue than the lottery, no significant difference is found between the two treatments. Finally, Schram and Onderstal (2006) present an experimental study that compares a winner-pay auction, an all-pay auction and a lottery in the case of heterogeneous preferences, but homogeneous endowments. They find that the all-pay auction performs better than the lottery mechanism, as predicted by the theory. In sum, all of these studies focus on the case of homogeneous endowments.

Our analysis is based on a theoretical framework where prizes are used as a means to finance public goods when agents have heterogeneous endowments which are private information. In this setting, an all-pay auction generates a higher expected total contribution than a lottery with an equal prize. In the all-pay auction, it is possible to identify a monotone equilibrium such that contributions are strictly increasing in the endowment. The equilibrium of the lottery is instead completely symmetric, as in Morgan (2000), with all agents contributing the same amount independently of their endowment.

In our experiment, we test the following theoretical predictions. First, incentive-based mechanisms should outperform the voluntary contribution mechanism in terms of net contributions (after taking into account the cost of prizes). Second, the total revenue of the all-pay auction should be higher than that of a lottery with an equal prize. Third, absolute contributions should not depend on income in the lottery, whereas they should rise with income in the all-pay auction. As a consequence, individual contributions should be higher in the lottery than in the all-pay auction at the lower end of the income distribution, while the opposite should hold at the upper end of the income distribution.

The main findings of the analysis can be summarised as follows. In all mechanisms average contributions are generally higher than theoretical predictions and tend to converge towards the predicted values over successive rounds. The voluntary contribution mechanism replicates the behavioural patterns observed in similar experiments under income homogeneity or heterogenous incomes and complete information. The introduction of a prize as an incentive has significant effects on contributions: the lottery and, to a lesser extent, the all-pay auction perform better than voluntary contribution in terms of public good provision after accounting for the cost of prizes. Comparing the prize-based mechanisms, contributions are significantly higher in the lottery than in the all-pay auction, contrary to the theoretical predictions. Focusing on the behaviour of individual income types, absolute contributions rise with income in all treatments, although more steeply in the prize-based mechanisms, so that relative contributions are generally similar across income types. In terms of relative performance, the lottery does better than the other mechanisms for all income types. At the individual level, subjects choose zero contributions in the all-pay auction about three times as often as in the lottery.

The paper is structured as follows. Section 2 presents the theoretical background of the analysis. Section 3 describes the experimental design and the theoretical predictions to be tested. Section 4 presents the results. Section 5 concludes with a discussion of the main findings and implications of the analysis.

2 Theoretical Background

In this section we present the main theoretical predictions for the performance of single-prize all-pay auctions and lotteries, as fund-raising mechanisms for the private provision of public goods, under the assumptions of income heterogeneity and incomplete information about income levels. The analysis is based on the framework presented in Faravelli (2007).

Consider a public good game with n risk neutral players. Each player i is assumed to have income z_i, which is private information. Incomes are drawn independently of each other from the distribution function $F(z)$ on the interval $[\underline{z}, \bar{z}]$. $F(z)$ is common knowledge and has a continuous and bounded density $F'(z)$. Each player has to decide how much of his endowment to contribute to the public good, knowing that total contributions to the public good are multiplied by a parameter $\alpha \in (1, n)$ and shared equally among all the agents. The cost of contributing to the public good exceeds the marginal return of investing in it. Therefore, the unique Nash equilibrium is to contribute nothing, although it is socially optimal to contribute all the endowment.

Suppose that the fund-raiser has access to an amount Π. The fund-raiser moves first. He can either use Π to provide the public good or organise either a lottery or an all-pay auction in which the winner is awarded a prize equal to Π. Then the agents choose their contributions in order to maximise their utility (expected utility in the case of an all-pay auction), given the other players' contributions and the value of Π. If the fund-raiser spends all his budget Π to provide the public good then the payoff of of player i with endowment z_i who contributes g_i is given by

$$z_i - g_i + \frac{\alpha}{n}(\Pi + g_i + G_{-i})$$

where G_{-i} is the sum of the contributions of all the other players. If the fund-raisers uses Π as a prize, player i's payoff will be

$$z_i - g_i + E[\Pi, g_i, g_{-i}] + \frac{\alpha}{n}(g_i + G_{-i})$$

where g_{-i} represents the vector of the individual contributions of all the other players. $E[\Pi, g_i, g_{-i}]$ is the expected prize of player i given all the other players' contributions. In the lottery, player i wins the prize with probability $\frac{g_i}{g_i+G_{-i}}$, which is the number of tickets he bought divided by the total number of tickets. In the all-pay auction he wins if and only if his contribution is higher than all the other agents' contributions.

The main results for the different contribution mechanisms can be summarised as follows.[4] In the voluntary contribution mechanism the Nash equilibrium is to contribute nothing. At interior solutions,[5] the lottery has a symmetric pure strategy equilibrium (as in Morgan, 2000) where every player contributes the same amount

$$g^{LOT} = \frac{n-1}{n(n-\alpha)}\Pi \tag{1}$$

and the total contribution is

$$G^{LOT} = \frac{n-1}{n-\alpha}\Pi$$

Total contribution is higher than the cost of the prize. If public good provision is socially desirable the lottery provides positive net revenues. The all-pay auction, at an interior solution for all players, has a symmetric pure strategy equilibrium given by

$$g^{APA}(z) = \frac{n}{n-\alpha}F(z)^{n-1}\Pi \tag{2}$$

[4]See Faravelli (2007) for the proofs of the following results.

[5]We focus on the case in which constraints are non-binding for all agents.

Note that contributions in equilibrium are strictly increasing in the endowment. $F(z)^{n-1}\Pi$ represents the expected prize of a player with endowment z, when all players play according to the same strictly increasing strategy. Total expected contribution in the all-pay auction is given by

$$E[G^{APA}] = \frac{n}{n-\alpha}\Pi$$

The above expression is strictly greater than both the cost of the prize and the total contribution under the lottery. A lottery can be thought of as a stochastic all-pay auction, where the higher noise results in lower revenue (see Tullock, 1980).

Note that the equilibrium strategy function described by expression (2) can be rearranged as $g^{APA}(z) = \frac{1}{1-\frac{\alpha}{n}} F(z)^{n-1}\Pi$. This is the sum of a convergent series with reason $\frac{\alpha}{n}$ and can be written as

$$g^{APA}(z) = F(z)^{n-1}\Pi \sum_{m=0}^{\infty} (\frac{\alpha}{n})^m$$

The standard result in all-pay auctions is the total dissipation of the rent. In this case each individual bids more than the expected prize because of the marginal return of investing in the public good, which is equal to $\frac{\alpha}{n}$.

3 Experimental Design

Our experimental design follows Morgan and Sefton (2005) and, more closely, Orzen (2005), while introducing income heterogeneity and incomplete information about the income of other subjects. We considered three different treatments: a voluntary contribution mechanism (VCM), a lottery (LOT) and an all-pay auction (APA). We ran three sessions for each treatment, with sixteen subjects participating in each session, for a total of 144 subjects. Each session consisted of 20 rounds.

3.1 The Baseline Game

The baseline game is a standard linear public good game, with the introduction of income heterogeneity and incomplete information about income levels. At the beginning of each session the sixteen subjects were randomly and anonymously assigned an endowment of either 120, 160, 200, or 240 tokens.[6] The subjects were informed that in each round they would receive the same endowment as determined at the beginning of the session.

[6]Concentrating on a small number of possible endowments allowed us to analyse the effects of income heterogeneity in a controlled and simple setting, while providing several observations on the same income type.

Incomplete information about incomes was introduced by using a matching procedure similar to the *strangers* condition used in Andreoni (1988). At the beginning of each round, subjects were randomly and anonymously rematched in groups of four players. Therefore, in each round subjects did not know the identity and the endowment of the other three members of their group. They only knew that the endowment of each of the other group members could be either 120, 160, 200, or 240 tokens with equal probabilities.

Group matching for each of the twenty rounds was determined randomly before the beginning of the experiment in the following way. Four pools of four subjects were formed, each containing the four different income types (120, 160, 200, 240). Each of the four groups was formed by randomly drawing one subject from each pool. As a consequence, within every group each member could have an endowment of 120, 160, 200, or 240 tokens with equal probability.[7] Having formed the four groups for each round in this way, the same sequence of group matchings for the twenty rounds was used in each session of all three treatments.

In each round, every subject had to allocate entirely a given endowment between two accounts. The language used in the instructions did not refer to contributions or public goods, but asked subjects to allocate tokens to either an "individual account" or a "group account". We set $\alpha = 2$ and, with $n = 4$, the return from the group account was $\frac{\alpha}{n} = 0.5$. In order to avoid decimals, returns from both accounts were multiplied by two. Therefore, a subject received 2 points for each token he allocated to the individual account, while he received 1 point for each token allocated by him or by any other member of his group to the group account.

3.2 Treatments

The three treatments differed in the way prizes (extra points) could be earned by the subjects. In VCM, 120 tokens were exogenously allocated by the experimenter to the group account in each round, independently of the subjects' choices, thus implying that each member of the group received 120 points as a bonus. In LOT, a subject received a lottery ticket for each token he allocated to the group account. At the end of each round the computer randomly selected one ticket among all those purchased by the members of the group, and the owner of the selected ticket won the prize of 240 points. In case no tokens were allocated to the group account, the winner of the prize was selected randomly among the four members of the group. In APA, in each round the member of the group who allocated the highest amount to the group account won the prize of 240 points. In case of ties between two or

[7] Note that in every round there were four subjects for each of the four possible endowments, so that the average endowment was 180 tokens.

more group members, the winner was determined randomly by the computer. Note that the four mechanisms imply the same financial commitment for the fund-raiser: allocating 120 tokens to the group account in VCM is equivalent to paying a prize of 240 points in APA or in LOT.

3.3 Procedures

In each session, the subjects were randomly assigned to a computer terminal at their arrival. To ensure public knowledge, instructions were distributed and read aloud (see Appendix A for the instructions). Moreover, to ensure understanding of the experimental design, sample questions were distributed and the answers privately checked and, if necessary, individually explained to the subjects.

At the end of each round, the subjects were informed about their payoffs from the group account, the individual account and the prize (or bonus in VCM). At the end of the last round, subjects were informed about their total payoff for the twenty rounds expressed in points and euros. They were then asked to answer a short questionnaire on the understanding of the experiment and socio-demographic information, and were then paid in private using an exchange rate of 1000 points per euro. Subjects earned 12.25 euros on average for sessions lasting about 50 minutes, including the time for instructions. Participants were mainly undergraduate students of Economics and were recruited through an online system. The experiment took place in May 2006 at the Experimental Lab of the University of Milan Bicocca. The experiment was computerized using the zTree software (Fischbacher, 1999).

3.4 Predictions

In this experimental design, within each group every subject can have one of four possible endowments with equal probability. Compared with the model described in Section 2, where each player's endowment is drawn from a continuous distribution function, it is easy to see that the equilibrium in VCM will still be to contribute nothing. Similarly, LOT is characterised by the same equilibrium as described by equation (1), and the same total contribution. This is because the best response function of a player in the lottery game is independent of income, as in Morgan (2000). The equilibrium is instead slightly modified in APA, although qualitatively unchanged, given that the pure strategy equilibrium, as described by equation (2), depends on the continuity of endowment distribution.

In Appendix B we consider an all-pay auction in a linear public good game where each player's endowment is drawn from a discrete distribution function, under the assumption of *complete* information. We solve the game for N players, who can have any possible endowment, and for any positive level of prize. We show that when the prize is not too "high", only mixed strategy equilibria exist. The equilibrium for a subject under *incomplete* information about the incomes of other players consists of a randomisation over the mixed strategies he would play in all the possible group matchings he faces, according to their corresponding probabilities.[8]

The total expected contribution in APA for the mixed strategy equilibrium under a discrete income distribution is lower than that for the pure strategy equilibrium under a continuous income distribution. This loss of revenue results from the discontinuity in the possible endowments: a subject with an endowment higher than the lowest one will face opponents with a strictly lower endowment with positive probability. In this case he will not have any incentive to bid more than the highest of his opponents' endowments. Nevertheless, despite the lower expected revenue, all the theoretical predictions described in Section 2 under a continuous distribution of endowments are qualitatively unchanged.

Table 1 presents the predicted contributions for the experimental design, both in absolute and relative terms, for each income type and on average. Average contributions for both prize-based mechanisms are higher than the predicted contribution in VCM, which is zero. They are also higher than the average provision in VCM, where an amount equivalent to the cost of the prize is used to directly finance the public good, resulting in an average provision of 30 tokens per subject. The average absolute contribution in APA (51 tokens) is higher than in LOT (45 tokens). Note also that in LOT the predicted absolute contributions are independent of income levels (25% in relative terms on average). Predicted contributions are instead steeply increasing in the endowment in APA, both in absolute and relative terms.

Summing up, the main hypotheses to be tested are as follows:

Hypothesis 1 (Absolute Efficiency): Both LOT and APA outperform VCM not only in terms of gross contributions, but also after taking into account the cost of the prize.

Hypothesis 2 (Relative Efficiency): The total contribution to the public good is higher in APA than in LOT.

Hypothesis 3 (Individual income types): Individual contributions do not depend on income in LOT, while they increase with income in APA. Contributions are therefore higher in LOT

[8]For instance, a subject endowed with 120 tokens could be grouped with three other subjects with his same endowment, or with one subject with 160 and two others with 200 tokens, and so on.

Table 1: Theoretical predictions: absolute and relative contributions

Treatments	Incomes 120	160	200	240	Average 180
			Absolute contributions		
VCM	0	0	0	0	0
LOT	45	45	45	45	45
APA	5	28	68	102	51
			Relative contributions		
VCM	0	0	0	0	0
LOT	38	28	23	19	25
APA	4	18	34	43	28

Note: contributions are rounded to the nearest integer. Relative contributions are expressed as a percentage of the endowment.

than in APA for low-income types, while the opposite holds for high-income types.

4 Results

This section presents the experimental results. We start with a descriptive analysis of the main features of the data for the three treatments. Next, we examine the replicability of sessions within each treatment, the effects of repetition over rounds, and the dependence of individual observations within sessions. We then present formal tests of the theoretical predictions, considering first average contributions over all subjects and then contributions by individual income types.

4.1 Overview

Figures 1-3 present an overview of average relative contributions (as a percentage of the endowment) over rounds for each session of the three treatments. Table 2 reports relative contributions obtained by averaging over all subjects within each session for all 20 rounds and for the following sub-sets of rounds: 1st, first 10, last 10, and 20th.

The results for VCM sessions are similar to those generally obtained in public good experiments with homogeneous incomes. Average contributions to the group account are substantially higher than the equilibrium prediction of zero throughout the twenty rounds, but display a clear downward trend over successive rounds (Figure 1). Averaging over all sessions and subjects, individual contributions are 21.6% over the 20 rounds, falling from 35.1%

Table 2: Average individual contributions: by session and rounds

	Rounds				
Session	1-20	1	1-10	11-20	20
VCM 1	18.6	25.7	20.6	16.6	7.7
VCM 2	26.3	32.4	28.9	23.7	11.2
VCM 3	19.9	47.1	28.9	10.9	5.7
Average	21.6	35.1	26.1	17.1	8.2
LOT 1	42.1	38.5	45.3	38.9	39.9
LOT 2	46.2	48.5	50.0	42.3	39.3
LOT 3	52.7	45.7	52.8	52.5	41.8
Average	47.0	44.2	49.4	44.6	40.3
APA 1	41.8	51.8	46.0	37.7	52.4
APA 2	40.7	46.8	45.3	36.1	29.2
APA 3	36.2	45.3	38.5	33.9	30.3
Average	39.6	48.0	43.2	35.9	37.3

Note: contributions to the public good are expressed as a percentage of the endowment.

in the first round to 8.2% in the last round, and from 26.1% in rounds 1-10 to 17.1% in rounds 11-20. The same pattern of positive but declining contributions is observed in each of the three VCM sessions, and a clear tendency to converge to a common level is observed in the final rounds.

Average contributions for LOT sessions are also systematically higher than the predicted contribution of 25%, and remain virtually constant over time, except for a slight decline in the final rounds (Figure 2). Average contributions over all sessions, subjects and rounds are 47%, falling from 44.2% to 40.3% over the 20 rounds, and from 49.4% to 44.6% between the first and the second half of the session. The declining pattern is not observed in all sessions and, although the three sessions converge to a common level in the final round, a relatively high variability across sessions is observed in rounds 11-20.

In APA sessions, average contributions are larger than the predicted contribution of 28%, and display a slight decline over rounds (Figure 3). All sessions start with relatively high contributions, but tend to converge to the theoretical prediction within the first ten rounds. The average contribution is 39.6% over the twenty rounds, falling from 43.2% in rounds 1-10 to 35.9% in rounds 11-20. Each of the three APA sessions displays the same pattern of declining contributions, and the profiles are very similar.

Figure 1: Average contributions over time: VCM

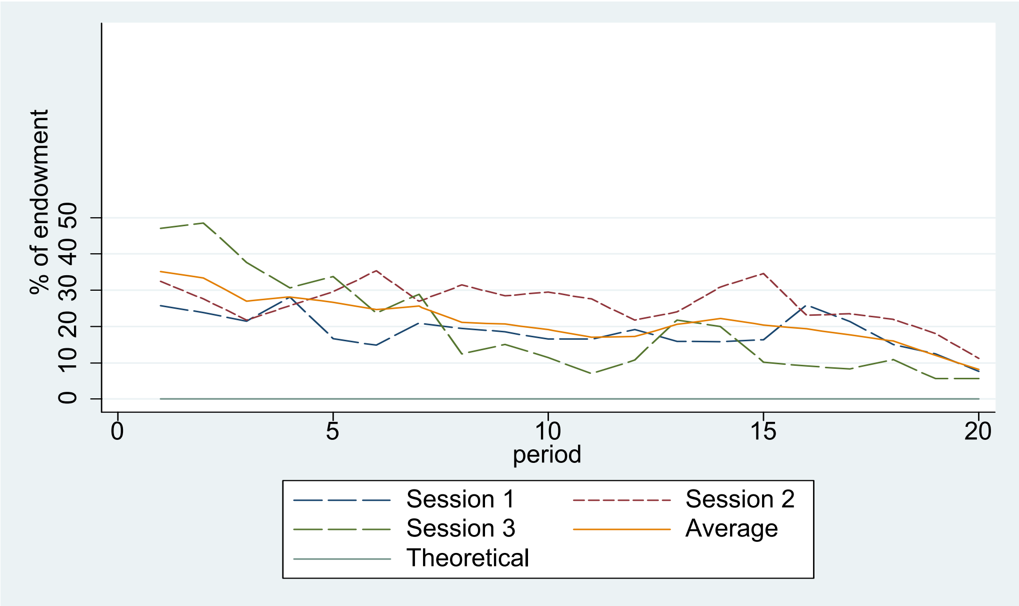


Figure 2: Average contributions over time: LOT

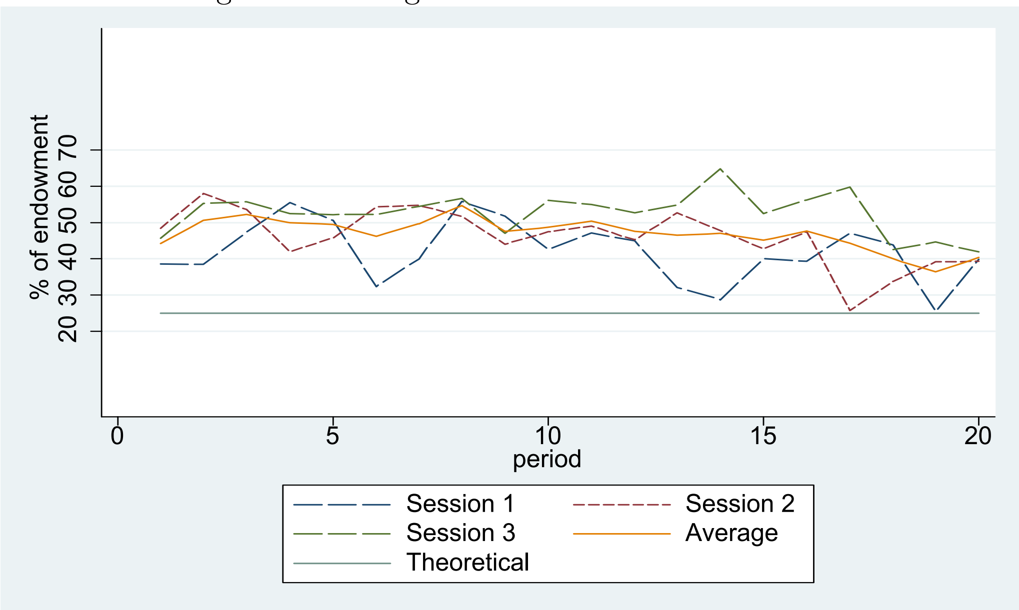
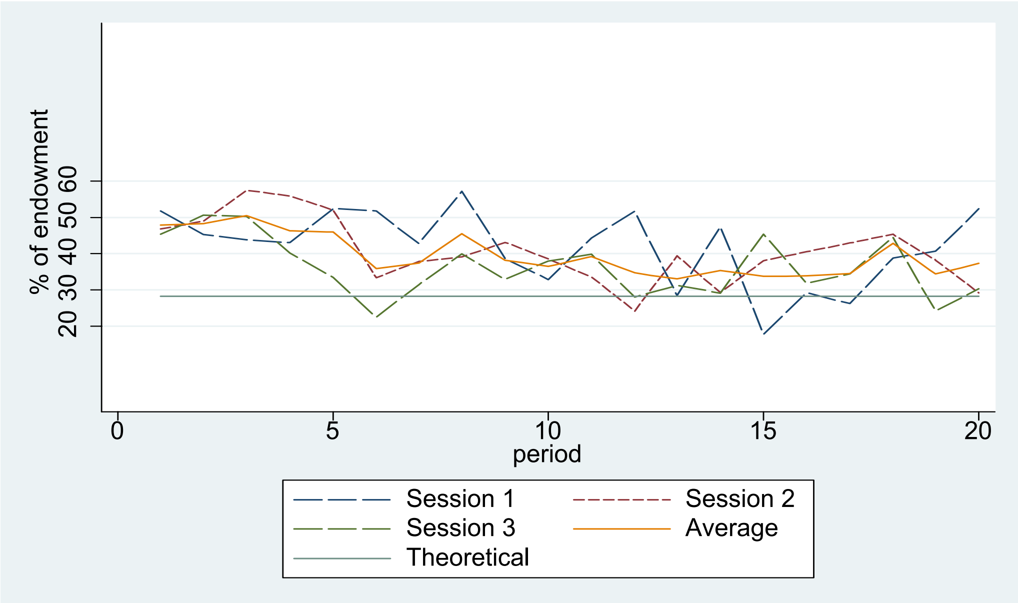


Figure 3: Average contributions over time: APA



4.2 Replicability, Repetition, Sectional Dependence

The descriptive analysis of session-level data indicates that contributions for individual sessions within each treatment are qualitatively similar in terms of both average levels and dynamics over rounds. We provide formal tests for the hypothesis of replicability of session results within treatments. Table 3 presents Kruskal-Wallis test statistics for the null hypothesis that median contributions are equal across the three sessions within each treatment, focusing on the same sub-sets of rounds as in Table 2. Focusing on the whole session (rounds 1-20) or the last round, the results indicate that the null hypothesis cannot be rejected for all treatments. We therefore conclude that the three sessions can be pooled and the analysis is carried out on observations for 48 individuals for each of the three treatments.

The analysis of session-level data also indicates that there are substantial changes in contributions over successive rounds (repetition effects). Contributions tend to fall over rounds, generally converging towards theoretical predictions, in all treatments except for LOT, where the tendency to converge is less marked. This could suggest that excessive contributions may be due to the fact that subjects are learning how to behave rationally. We provide formal tests for the effects of repetition on contributions. Table 4 presents results of Wilcoxon signed-rank tests for the hypothesis that median contributions are the same across selected pairs of rounds within each treatment.

Table 3: Tests for replicability of sessions

	Rounds				
Treatment	1-20	1	1 - 10	11-20	20
VCM	4.93	5.83	4.21	6.43	0.75
	(0.08)	(0.05)	(0.12)	(0.04)	(0.69)
LOT	2.16	1.08	0.89	3.68	0.26
	(0.34)	(0.58)	(0.64)	(0.16)	(0.88)
APA	1.90	0.50	1.67	0.99	2.83
	(0.39)	(0.78)	(0.43)	(0.61)	(0.24)

Note: the table reports Kruskal-Wallis test statistics for the null hypothesis that median contributions are equal across the three sessions within each treatment. P-values (in brackets) are based on the χ^2 distribution with 2 degrees of freedom.

Table 4: Tests for repetition effects

	Rounds				
Treatment	1 vs 10	10 vs 20	1 vs 15	5 vs 20	1 vs 20
VCM	3.04	2.96	3.26	4.11	5.54
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
LOT	-1.42	1.65	-0.04	1.23	0.99
	(0.16)	(0.10)	(0.97)	(0.22)	(0.32)
APA	2.04	0.16	1.95	-0.31	1.76
	(0.04)	(0.87)	(0.05)	(0.76)	(0.08)

Note: the table reports normalized Wilcoxon signed-rank test statistics for the hypothesis that median contributions are the same in the two rounds indicated in the column headings. P-values (in brackets) refer to two-sided tests based on the standard normal distribution.

Irrespective of the time horizon considered, decreases in contributions are significant in VCM. LOT does not display any significant round effect, whereas in APA the differences are significant between rounds 1 and 10 (p-value 0.04) and marginally significant between rounds 1 and 20 (p-value 0.08). We conclude that repetition affects different mechanisms in different ways, so that comparisons between treatments, and between actual and predicted contributions within treatments, cannot focus on a single round but should consider alternative sub-sets of rounds in order to take into account the different role played by repetition in each treatment.

In order to go beyond descriptive analysis and provide formal tests of the theoretical predictions we need to define the appropriate unit of analysis (subject, group, session). It is important to note that, because of repetition, subject-level observations within each session and round might be dependent, given that (in rounds beyond the first) subjects have interacted in previous rounds. In addition, because of the random rematching mechanism (at the beginning of each round subjects are randomly and anonymously rematched in groups of four people), independence could also be violated for group-level observations. If the dependence of subject-level observations due to interactions in earlier rounds was relevant, inference would have to be based on session-level observations (see e.g. Orzen, 2005).

However, the characteristics of the experimental design are such that the dependence across individual observations can be considered negligible. First, at the end of each round subjects only learn about the total contribution of other group members, so that it is difficult for them to infer individual absolute contributions. Second, since subjects do not know the endowments of other group members, it is even more difficult for them to infer other subjects' relative contributions (e.g. an absolute contribution of 120 could be a relative contribution of 50% as well as 100%, depending on the endowment of the other subject). Third, the number of players within each session (sixteen) is sufficiently large, so that subjects know that there is a relatively small probability of interacting with the same subjects as in the previous round. This further reduces the motivation to reciprocate in successive rounds, thus weakening the possible dependence across individual observations.

We also investigated the issue at the empirical level, by considering Spearman rank correlation tests for the null hypothesis of independence between the contributions of each subject and the average contributions of the subjects who were in his group in the previous round. The test statistics, based on 16 individual observations for each session and each round, are significant at the 5 per cent level in only about 15 percent of the cases. In addition, within the significant cases, 30% of the correlation coefficients are positive and 70% are negative, indicating that there is no systematic pattern in the relationship between each subject's contribution and those of his past group members. As a result, considering both the features of the experimental design and the results of the Spearman tests, we conclude that the dependence across individuals can be considered negligible. Hence, in the following, we use subjects as the unit of analysis (see Morgan and Sefton, 2000, for a similar approach).

4.3 Comparison between Treatments: Total Contributions

In order to compare the relative performance of the different funding mechanisms, Figure 4 displays average individual contributions over rounds for each treatment.[9] The introduction of prizes has a substantial effect on individual contributions. Average contributions in LOT are more than twice as large as in VCM over the 20 rounds, and about five times as large in the final round. Average contributions in APA are almost twice as large as in VCM over the 20 rounds and almost five times as large in the final round.

It is important to observe, however, that VCM is not directly comparable with the incentive-based treatments in terms of individual contributions. In order to make the results comparable we must either consider contributions *net of the cost of prizes* in the incentive-based mechanisms or, equivalently, refer to overall *provision* (i.e. also take into account public provision in VCM). Public provision in VCM accounts for 120 tokens per group, corresponding to about 17 percentage points per subject in terms of relative contributions. Figure 5 displays average public good *provision* over rounds, providing the appropriate reference for comparing incentive-based mechanisms with the benchmark VCM.

Interestingly, when we compare the treatments in terms of overall provision (or, equivalently, contributions *net* of the cost of prizes), the comparison of the incentive-based mechanisms relative to VCM is not as clear-cut as before. While LOT systematically outperforms VCM (with the only exception of the first round), APA provision is very close to VCM, except for the last rounds where the two profiles diverge. Averaging over all rounds, relative to VCM, public good provision in LOT is about 20 per cent higher and 3.5 per cent higher in APA.

[9] The graphs confirm that repetition strongly affects each treatment in different ways. Both incentive-based mechanisms display high contribution levels in initial rounds, but their dynamics in the following rounds differ. While in the first ten rounds of APA contributions decline markedly, in the lottery they remain virtually unchanged. Thereafter, contributions fall somewhat in LOT, while they remain constant in APA. As a consequence, focusing on final rounds only, LOT and APA converge to very similar contribution levels. VCM contributions are on a downward trend and are systematically lower than those of the other two mechanisms.

Figure 4: Average individual contributions: all treatments

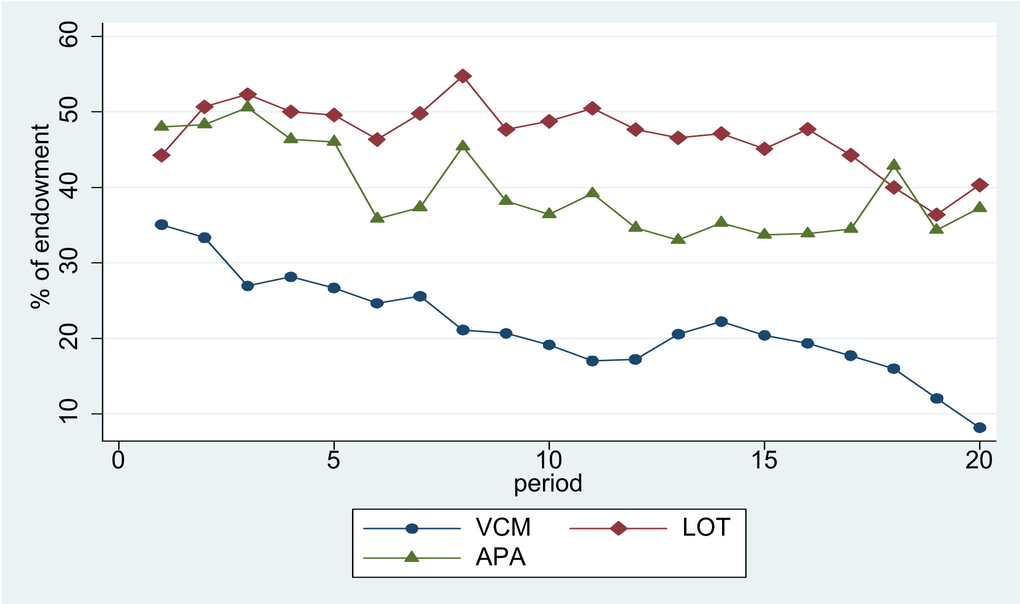
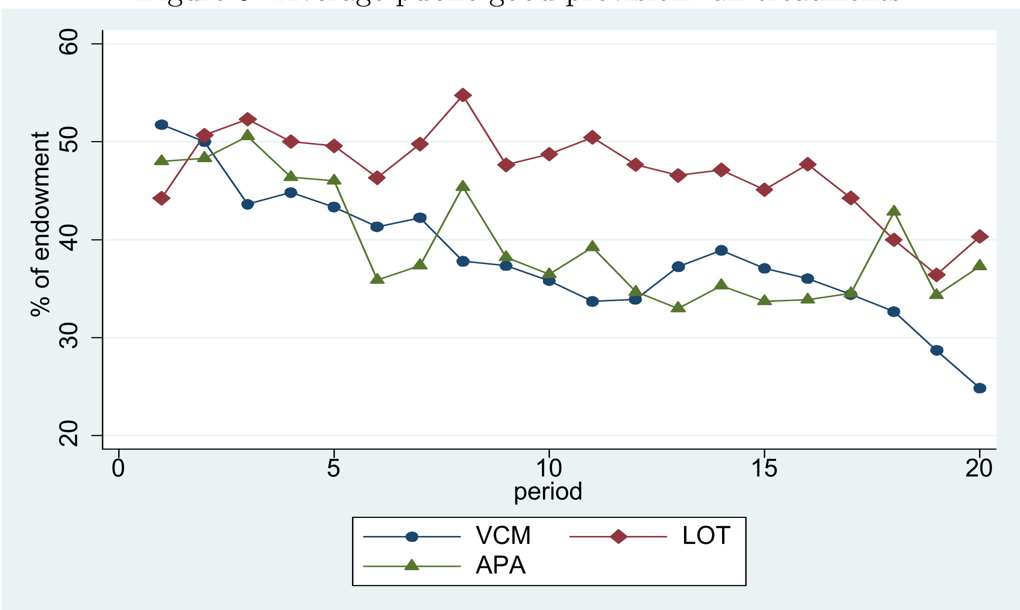


Figure 5: Average public good provision: all treatments



The informal evidence presented in Figure 5 is examined further in Table 5, presenting results of Wilcoxon rank-sum tests of the null hypothesis that median public good provision is the same across treatments. The first two tests compare each of the incentive-based mechanisms with the benchmark VCM. The next compares APA with LOT. Given that our model predicts the direction of departure from the null hypothesis, we use the relevant one sided-tests.

Table 5: Tests of equality between treatments: All subjects

	Rounds				
Treatments	1-20	1	1 - 10	11-20	20
LOT - VCM	4.55	-3.10	3.43	4.75	5.22
	(0.00)	(1.00)	(0.00)	(0.00)	(0.00)
APA - VCM	1.88	-1.55	1.12	1.52	4.75
	(0.03)	(0.94)	(0.13)	(0.06)	(0.00)
APA - LOT	-4.09	1.52	-2.84	-4.39	-0.92
	(1.00)	(0.06)	(1.00)	(1.00)	(0.82)

Note: the table reports Wilcoxon rank-sum tests (normalized z-statistics) for the hypothesis that the median of the difference between individual relative provisions to the public good in the given two treatments is zero. P-values (in brackets), based on the standard normal distribution, refer to one-sided tests as predicted by the theory.

The test statistics are positive and highly significant at all time horizons (except for round 1) in the comparison between LOT and VCM. APA provision also significantly outperforms VCM, although the significance level is quite variable across sub-samples, owing to the different effects of repetition in the two treatments.

> **Result 1**: Both the lottery and the all-pay auction are more effective than voluntary contribution in funding public goods.

The comparison between LOT and APA indicates that not only APA does not perform better than LOT, but also public good provision is statistically higher in LOT than in APA, contrary to the model's predictions.

> **Result 2**: The lottery is more effective than the all-pay auction in funding public goods.

It is interesting to observe that incentive-based funding mechanisms are generally efficient in covering the cost of the prize. Averaging over all sessions and rounds for each treatment, group contributions cover the cost of the prize in 96.3 per cent of the cases in LOT and 88.3 per cent of the cases in APA. Thus the lottery outperforms the all-pay auction also in terms of financial efficiency.

4.4 Comparison between Treatments by Income Level

So far we have considered contributions by taking averages over all subjects, thus abstracting from differences across individuals characterized by different income levels. The theory, however, provides predictions for the contributions of each income type (see Table 1). In this section we focus explicitly on income heterogeneity. We first examine whether individuals with different incomes behave as predicted by the theory and how over-contribution is related to income levels. Next, we consider how different funding mechanisms compare at different ends of the income distribution.

Table 6 and Figures 6 and 7 provide a description of the relationship between contributions and income levels. In all treatments absolute provisions rise with income levels, so that relative provisions are generally relatively similar across income types. Over-contributions in VCM are observed for all income types, and rise slightly with income in absolute terms. In LOT all income types over-contribute and, contrary to the theoretical predictions, absolute contributions rise almost linearly with income. In APA, absolute contributions rise with income, although not as steeply as predicted by the theory. As a consequence, the three lowest-income types over-contribute, while the contributions of subjects with the highest income (240) are very close to the theoretical prediction.

> **Result 3**: Absolute contributions are weakly related to income in VCM, and steeply increasing in income in both LOT and APA.

Figures 8 and 9 provide a comparison of absolute and relative provision in the three treatments by income levels.[10] Interestingly, the lottery outperforms both other mechanisms for all income types. Public good provision in APA is higher than in VCM for income types 160 and 240, but the opposite holds for the lowest income type, indicating that prizes in contests provide relatively less effective incentives for poorer individuals.

Table 7 presents results of Wilcoxon rank-sum tests of the null hypothesis that median public good provision is the same across treatments, when considering separately each income type. The results indicate that LOT performs significantly better than VCM for all income types except the lowest. LOT also performs significantly better than APA for incomes 120 and 200. APA does significantly better than VCM only for income type 160.

> **Result 4**: The lottery outperforms the other funding mechanisms for all income types.

[10] Note that in order to ensure comparability we assumed that in VCM the 120 tokens of public provision can be attributed to each income-type on the basis of equal income shares (about 17%).

Table 6: Average relative contributions: by endowment and rounds

		Rounds				
Endowment	Predicted	1 - 20	1	1 - 10	11 - 20	20
VCM-120	0.0	28.4	37.8	36.5	20.2	9.2
VCM-160	0.0	15.3	31.5	16.3	14.4	8.6
VCM-200	0.0	23.8	35.8	27.2	20.3	10.8
VCM-240	0.0	19.0	35.1	24.6	13.4	4.1
LOT-120	37.5	47.1	36.6	52.0	42.3	40.3
LOT-160	28.1	48.2	46.4	49.8	46.6	45.6
LOT-200	22.5	48.5	46.3	49.8	47.1	43.1
LOT-240	18.7	44.1	47.7	46.0	42.2	32.3
APA-120	4.2	35.6	49.3	35.7	35.6	37.6
APA-160	17.5	40.7	35.7	42.3	39.1	41.5
APA-200	34.0	40.8	53.8	46.4	35.1	36.9
APA-240	42.5	41.1	53.1	48.5	33.7	33.2

Note: contributions are expressed as a percentage of the endowment.

4.5 Comparison between Treatments at Individual Level

We finally consider the relative performance of the three mechanisms at the individual level. Figure 10 compares the cumulative distribution functions of relative contributions for the three treatments. The main difference between the two prize-based mechanism is that in APA subjects choose zero contributions about three times as often as in LOT (20.8 and 5.83 per cent, respectively). The cumulative distribution for APA lies above that for LOT only up to a relative contribution of 50 per cent, while the two distributions are virtually identical thereafter. Note also that, although average contributions in APA and VCM are relatively similar, the distributions of individual contributions in the two treatments are similar only for low relative contributions.

> **Result 5**: At the individual level, APA is characterized by a much higher fraction of zero contributions than LOT.

Figure 11 compares the cumulative distribution functions of relative contributions for the three treatments, considering individual income types. It is interesting to observe that the difference between APA and LOT in the frequency of low contributions is very pronounced for low income types, but it becomes less and less evident for higher income types. While in LOT subjects contribute almost uniformly irrespective of their income type, in APA low-income individuals contribute zero much more often, as predicted by the theory. Nevertheless, low relative contributions are much more frequent than predicted by the theory for high-income individuals.

Table 7: Tests of equality between treatments by income level

	Incomes			
Treatments	120	160	200	240
LOT - VCM	1.05	2.62	2.09	3.14
	(0.15)	(0.00)	(0.02)	(0.00)
APA - VCM	-1.05	2.62	0.00	1.32
	(0.85)	(0.00)	(0.50)	(0.09)
APA - LOT	-3.14	-1.57	-2.09	-0.52
	(0.00)	(0.06)	(0.98)	(0.70)

Note: the table reports Wilcoxon rank-sum tests (normalized z-statistics) for the hypothesis that the median of the difference between individual contributions in the given two treatments is zero. P-values (in brackets), based on the standard normal distribution, refer to one-sided tests as predicted by the theory.

Figure 6: Average absolute contributions by endowment: all treatments

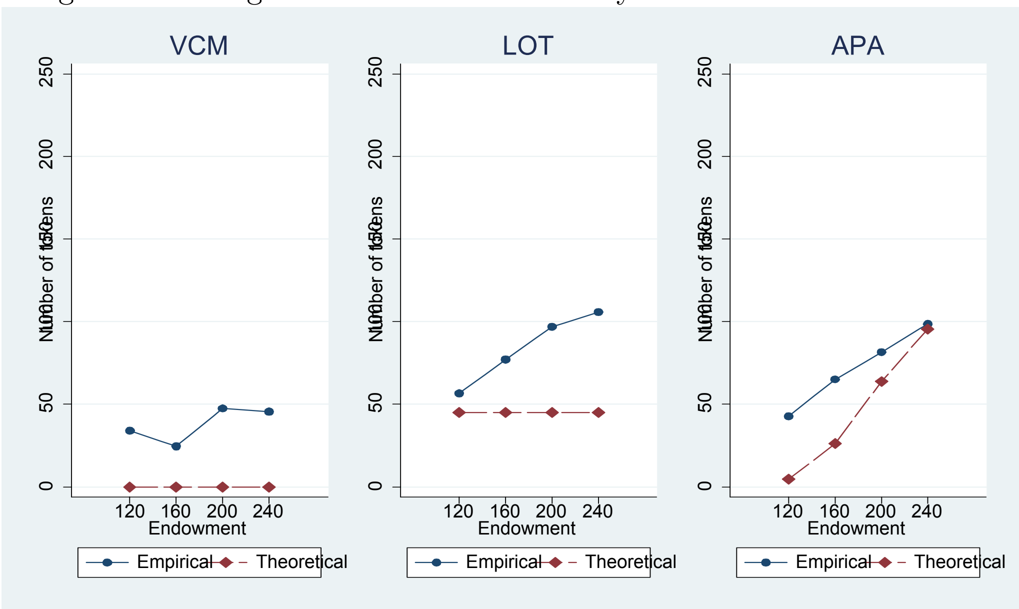


Figure 7: Average relative contributions by endowment: all treatments

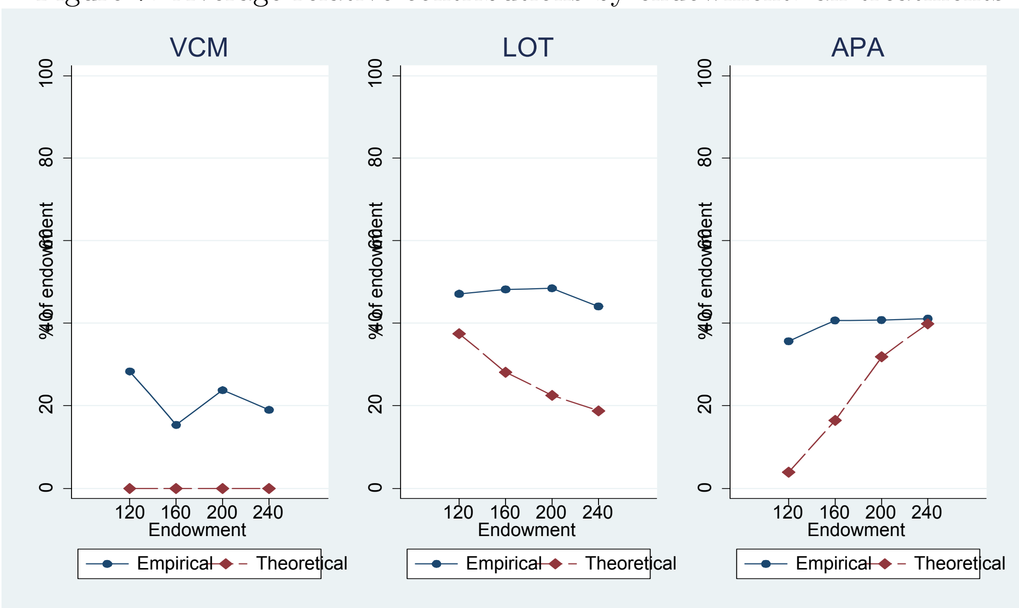


Figure 8: Average absolute provisions by endowment: all treatments

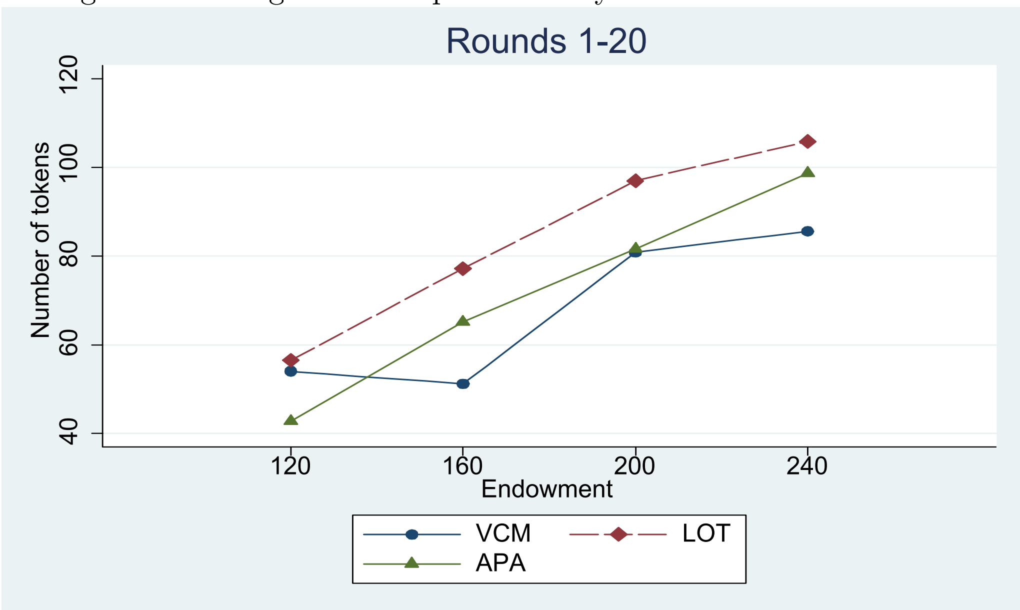


Figure 9: Average relative provisions by endowment: all treatments

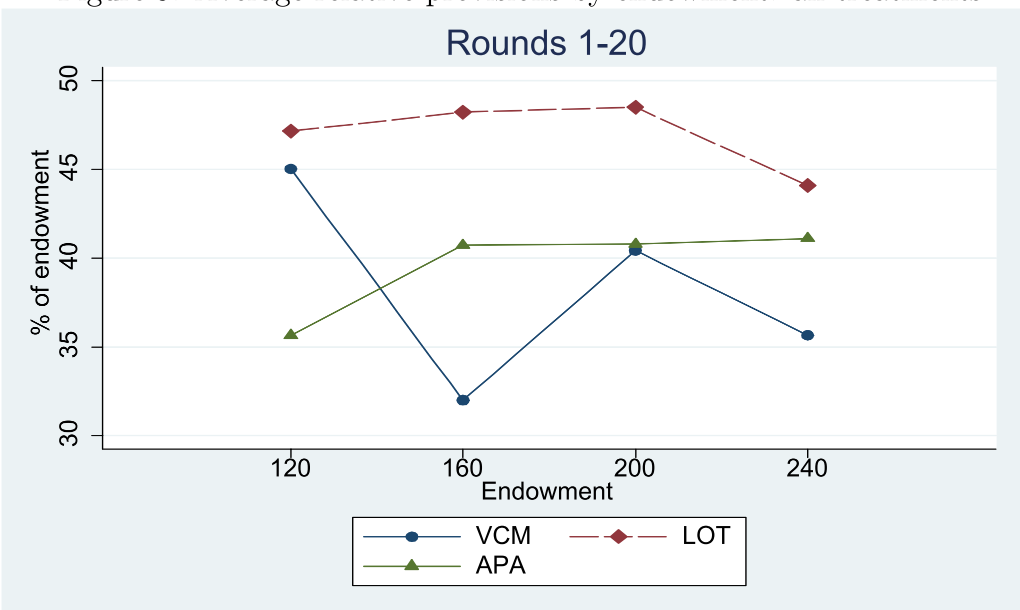


Figure 10: Cumulative distribution of contributions, by treatment

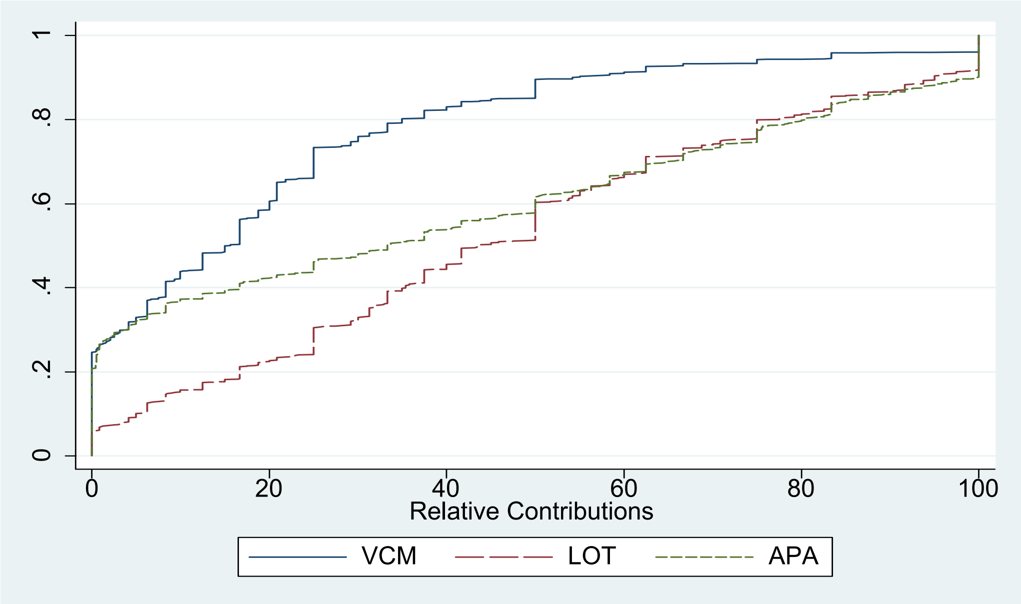
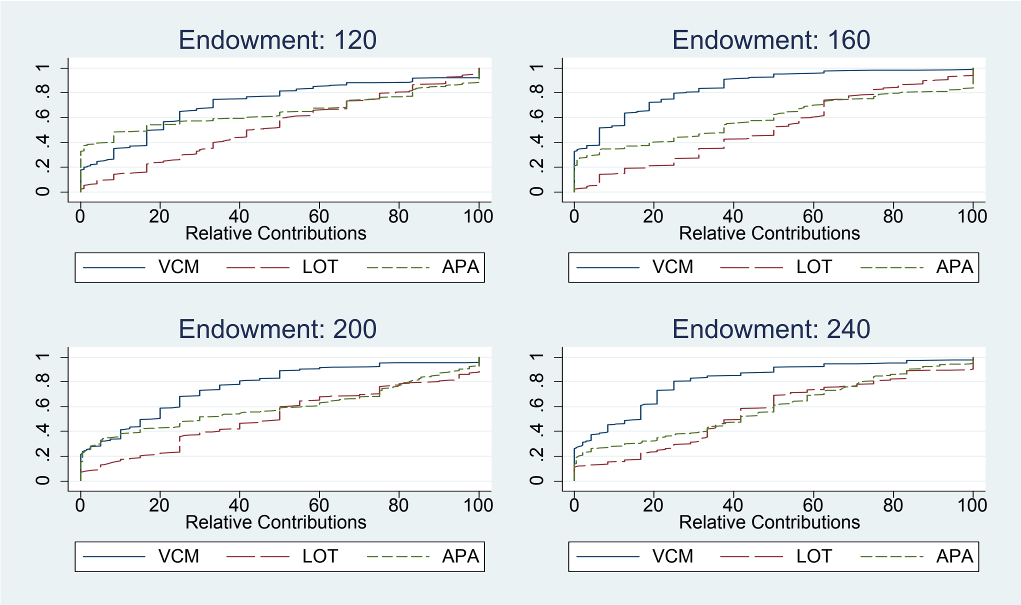


Figure 11: Cumulative distribution of contributions, by treatment and endowment



5 Conclusions

A number of experimental papers have analysed public good provision when incomes are heterogeneous. However, these studies have only explored voluntary contributions. The experimental literature on fund-raising mechanisms based on prizes has focused on the case of income homogeneity. To our knowledge, this paper is the first experimental investigation of the performance of incentive-based fund-raising mechanisms when subjects have heterogenous endowments which are private information. We compared a lottery and an all-pay auction, while also considering a voluntary contribution mechanism as a benchmark.

The results indicate that the introduction of a prize has sizeable effects on individual contributions relative to the VMC. Both the lottery and the all-pay auction outperform voluntary contributions after taking into account the cost of the prize. The lottery, in particular, systematically and significantly outperforms VCM. Averaging over all rounds, public good provision is 20 per cent higher in the lottery than in VCM. Provision in the all-pay auction is also higher than in VCM, but the difference is not significant in the earlier rounds of the sessions. This is an important result. It indicates that, in a setting where agents have heterogeneous incomes which are private information, prize-based fund-raising mechanisms can be an effective way of overcoming free riding.

The comparison between the incentive-based mechanisms indicates that, contrary to the theoretical predictions, contributions to the public good are significantly higher in the lottery than in the all-pay auction. This result suggests a number of possible interpretations. It could be argued that subjects are more familiar with lotteries than with all-pay auctions. As a consequence, they might tend to bid more conservatively in the latter. This intuition is supported by the finding that, at the individual level, subjects choose zero contributions in APA three times as often as in LOT. It could also be argued that subjects perceive the lottery as more fair than the all-pay auction. However, such arguments would not help explain the differences between our result and those in Orzen (2005) and in Schram and Onderstal (2006). The first study found no significant difference between the two mechanisms, focusing on homogeneous endowments and complete information. On the other hand, Schram and Onderstal (2006) focused on the case of symmetric endowments but heterogeneous preferences which are private information, finding that the all-pay auction raises higher revenues, as predicted by the theory.

Focusing on income heterogeneity, over-contributions are observed for all income types in VCM, and are slightly increasing with income in absolute terms. In the all-pay auction, absolute contributions rise with income, even though not as steeply as predicted by the theory. In the lottery, all income types over-contribute and, contrary to the theoretical predictions, absolute contributions rise linearly with income. The comparison of contributions across treatments indicates that the lottery outperforms both other mechanisms for all income types. This result indicates that, from a theoretical perspective, the completely symmetric equilibrium of a lottery game does not seem to properly describe the actual behaviour of subjects. Further, experiments on lotteries focusing on homogeneous endowments may be missing a crucial trait of the subject's behaviour.

There are several extensions for future research. A crucial point would be to investigate whether and how the specific features of the experimental designs can explain the differences between our results and the findings of other studies (see Orzen, 2005; Schram and Onderstal, 2006). Further, it would be interesting to develop a model that predicts a positive correlation between contributions and endowments in a lottery.

Appendix A: Instructions

[ALL TREATMENTS]

Welcome. Thanks for participating in this experiment. If you follow the instructions carefully and make good decisions you can earn an amount of money that will be paid to you in cash at the end of the experiment. During the experiment you are not allowed to talk or communicate in any way with other participants. If you have any questions raise your hand and one of the assistants will come to you to answer it. The rules that you are reading are the same for all participants.

General rules

There are 16 people participating in this experiment. At the beginning of the experiment each participant will be assigned randomly and anonymously an endowment of either 120, 160, 200, or 240 tokens with equal probabilities.

The experiment will consist of 20 rounds. In each round you will have the same endowment that has been assigned to you at the beginning of the experiment. In each round you will be assigned randomly and anonymously to a group of four people. Therefore, of the other three people in your group you will not know the identity and the endowment, that could be 120, 160, 200, or 240 tokens with equal probabilities.

How your earnings are determined

In each round you have to decide how to allocate your endowment between an INDIVIDUAL ACCOUNT and a GROUP ACCOUNT, considering the following information:

- for each token that you allocate to the INDIVIDUAL ACCOUNT you will receive 2 points.
- for each token allocated to the GROUP ACCOUNT (by you or by any other of the members of your group), every group member will receive 1 point.

[VCM]

In each round you will receive 120 bonus points.

At the end of each round the computer will display how many tokens you have allocated to the two accounts and how many points you have obtained from each of the two accounts and in total. At the end of the experiment the total number of points you have obtained in the 20 rounds will be converted in Euros at the rate 1000 points = 1 Euro. The resulting amount will be paid to you in cash.

[**LOT**]

In each round you can win a prize of **240 points** on the basis of the following rules. For each token allocated to the GROUP ACCOUNT you will receive a lottery ticket. At the end of each round the computer selects randomly the winning ticket among all the tickets purchased by the members of your group. The owner of the winning ticket wins the prize of 240 points. Thus, your probability of winning is given by the number of tokens you place in the GROUP ACCOUNT divided by the total number of tokens placed in the GROUP ACCOUNT by members of your group. In case no tokens are placed in the GROUP ACCOUNT, the winner of the prize is selected randomly among the four members of the group.

At the end of each round the computer will display how many tokens you have allocated to the two accounts and how many points you have obtained from each of the two accounts, from the prize, and in total. At the end of the experiment the total number of points you have obtained in the 20 rounds will be converted in Euros at the rate 1000 points = 1 Euro. The resulting amount will be paid to you in cash.

[**APA**]

In each round you can win a prize of **240 points** on the basis of the following rules. The member of your group who allocates the highest amount to the GROUP ACCOUNT is the winner of the prize. In case of ties among one or more group members, the winner is determined randomly.

At the end of each round the computer will display how many tokens you have allocated to the two accounts and how many points you have obtained from each of the two accounts, from the prize, and in total. At the end of the experiment the total number of points you have obtained in the 20 rounds will be converted in Euros at the rate 1000 points = 1 Euro. The resulting amount will be paid to you in cash.

Appendix B

In this appendix we study a linear public good game financed through an all-pay auction in which one prize is awarded, assuming that players have homogeneous preferences but heterogeneous endowments and information is complete. We solve the game for N players, who can have any possible endowment, and for any positive level of prize.

Consider N players and the set of endowments $Z = (z_1, ..., z_S)$ such that $0 < z_1 < \ldots < z_S$. Each player has an endowment which assumes a value from the set Z. Call $n[z_i]$ the number of players with endowment $z_i \in Z$ such that $\sum_{i=1}^{S} n[z_i] = N$. The players' endowments and their number are common knowledge. With no loss of generality, assume that $n[z_i] \geq 0$ for $1 \leq i \leq S-1$ and $n[z_S] \geq 1$. Players play a public good game in which each individual has to choose how much to contribute to the public good. At the same time they take part in an all-pay auction in which a prize is awarded to the agent who contributes the most. The bidders are risk-neutral and they all value the prize at $\Pi > 0$.

The payoff for a player with endowment z_i who contributes g_i is given by

$$\beta(z_i - g_i) + \beta E[\Pi, g_i, g_{-i}] + g_i + G_{-i}$$

where G_{-i} represents the sum of all other players' contributions and $1 < \beta < N$. Note that this expression is the same as the one used to define the payoff in Section 2, with $\beta = \frac{n}{\alpha}$. Note also that, in the experimental design presented in Section 3, $n = 4$ and $\alpha = 2$, so that $\beta = 2$.

We divide our analysis in two parts: the case where $n[z_S] > 1$ and where $n[z_S] = 1$.

More than One Player with the Highest Endowment

We study first the case in which $n[z_S] > 1$. Let us define the following equilibrium.

Definition 1 *Call type-symmetric equilibrium an equilibrium in which agents with the same endowment play according to the same strategy.*

There exist three possible scenarios: the prize level can be "high", "medium" or "low". In the next two propositions, we show that if and only if the prize level is "high" there exists a type-symmetric pure strategy equilibrium.

Proposition 1 *When $n[z_S] > 1$ and $z_S \leq \frac{\beta\Pi}{n[z_S](\beta-1)}$, there exists a type-symmetric pure strategy equilibrium in which players with endowment z_S contribute their full endowment, while if there are other agents with lower endowments they all contribute* 0.

Proof. If all players with endowment z_S contribute their full endowment each of them has an expected payoff of

$$\beta z_s + \frac{\beta\Pi}{n[z_S]} - (\beta - 1)z_S + G_{-i}$$

which is greater or equal than the payoff he could get from any other choice $g \in [0, z_S)$.[11]

If there are other players with lower endowments it is equally obvious that contributing 0 is for them a dominant strategy. ■

Proposition 2 *When $n[z_S] > 1$ and $\frac{\beta\Pi}{n[z_S](\beta-1)} < z_S$, there exist no type-symmetric pure strategy equilibria.*

Proof. In order to prove this it is enough to show that there exist no equilibria in which players with endowment z_S play according to the same pure strategy. The proof is in two parts.

i) Consider first the case in which $\frac{\beta\Pi}{n[z_S](\beta-1)} < z_S \leq \frac{\beta\Pi}{\beta-1}$. Suppose that players with endowment z_S contribute $g \in [0, z_S)$, then player i has an incentive to raise his own bid by an amount ε and win the prize. Equally, if all of them contribute z_S, then player i has an incentive to contribute 0.

ii) Consider now the case in which $z_S > \frac{\beta\Pi}{\beta-1}$. Notice first that any contribution $g > \frac{\beta\Pi}{\beta-1}$ is dominated by $g = 0$. Suppose that players with endowment z_S contribute $g \in [0, \frac{\beta\Pi}{\beta-1})$. Player i has an incentive to raise his own bid by an amount ε and win the prize. On the other hand if all of them contribute $g = \frac{\beta\Pi}{\beta-1}$, then player i has an incentive to deviate and contribute nothing. ■

If the prize level is "medium" only the agents with the highest endowment will submit non-zero bids.

Proposition 3 *When $\frac{\beta\Pi}{n[z_S](\beta-1)} < z_S < \frac{\beta\Pi}{\beta-1}$ and $n[z_S] > 1$ there exists a mixed strategy equilibrium in which:*

- *players with endowment z_S contribute their full endowment with probability p and with probability $1-p$ they choose their contribution from the distribution function $F(g) = (\frac{(\beta-1)g}{\beta\Pi})^{\frac{1}{n[z_S]-1}}$ on the interval $[0, a]$, such that $F(a) = 1 - p$, where $a < z_S$ and p is the unique solution to the following equation $\frac{1-(1-p)^{n[z_S]}}{n[z_S]p} = \frac{(\beta-1)z_S}{\beta\Pi}$;*
- *players with endowments lower than z_S contribute 0.*

[11] In all the proofs G_{-i} represents the sum of all other agents' contributions.

Proof. The proof is in five parts.

Let us first focus on the players with endowment z_S and show that, when they are the only active bidders, the candidate equilibrium is indeed an equilibrium.

i) Assume that all but one of the $n[z_S]$ players with endowment z_S choose their contribution from the distribution function $F(g) = (\frac{(\beta-1)g}{\beta\Pi})^{\frac{1}{n[z_S]-1}}$ on the interval $[0, a]$, where $0 < a < z_S$. Then the expected payoff of the remaining player i from contributing $g \in [0, a]$ is given by

$$\begin{aligned} & \beta z_S + \beta\Pi(F(g))^{n[z_S]-1} - (\beta-1)g + G_{-i} \\ = \quad & \beta z_S + G_{-i} \end{aligned}$$

which is independent of g.

Assume now that $n[z_S]-1$ players contribute their full endowment with probability p. Player i's expected prize from contributing z_S is given by

$$\beta\Pi \sum_{j=0}^{n[z_S]-1} \frac{1}{j+1} \binom{n[z_S]-1}{j} p^j (1-p)^{n[z_S]-j-1} \tag{3}$$

where $\binom{n[z_S]-1}{j} p^j (1-p)^{n[z_S]-j-1}$ represents i's probability of tying with j other players, while $\frac{\beta\Pi}{j+1}$ is his expected prize when he ties with j others. Applying binomial rules expression (3) can be rewritten as

$$\beta\Pi \frac{1-(1-p)^{n[z_S]}}{n[z_S]p}$$

and therefore player i's expected payoff from playing z_S is given by

$$\beta z_S + \beta\Pi \frac{1-(1-p)^{n[z_S]}}{n[z_S]p} - (\beta-1)z_S + G_{-i}$$

For this to be an equilibrium player i's expected payoff from contributing z_S must be equal to his expected payoff from choosing any $g \in [0, a]$, which means that

$$\begin{aligned} & \beta z_S + \beta\Pi \frac{1-(1-p)^{n[z_S]}}{n[z_S]p} - (\beta-1)z_S + G_{-i} \\ = \quad & \beta z_S + G_{-i} \end{aligned}$$

Therefore p must satisfy the following

$$\frac{1-(1-p)^{n[z_S]}}{n[z_S]p} = \frac{(\beta-1)z_S}{\beta\Pi} \tag{4}$$

ii) We are going to prove that there is a unique solution to equation (4). This equation can be rewritten as

$$1-(1-p)^{n[z_S]}=\frac{n[z_S](\beta-1)z_S}{\beta\Pi}p$$

Notice that the left hand side is concave while the right hand side is linear. Further, given the restrictions on z_S, it is the case that $1<\frac{n[z_S](\beta-1)z_S}{\beta\Pi}<n[z_S]$. When $p=0$ both sides of the equation are equal to zero. When $p=1$ the left hand side is equal to 1 while the left hand side is strictly greater than 1. Finally, notice that the slope of the left hand side when $p=0$ is $n[z_S]$, which is steeper than the right hand side. Therefore there must be a unique solution for $p\in(0,1]$.

iii) We want to show that a, such that $F(a)=1-p$, is strictly less than z_S. We will prove it by contradiction. Assume the opposite, then it should be the case that $F(z_S)\leq 1-p$. Given equation (4), the latter can be rearranged as

$$1-(1-p)^{n[z_S]}\leq n[z_S]p(1-p)^{n[z_S]-1} \tag{5}$$

When $p=0$ both sides are equal to 0. The first derivative of the left hand side is equal to $n[z_S]\,(1-p)^{n[z_S]-1}$, while the first derivative of the right hand side is $n[z_S]\,(1-p)^{n[z_S]-1}-(n[z_S]-1)n[z_S]p(1-p)^{n[z_S]-2}$. Notice that the former is strictly greater than the latter for any p on the interval $(0,1]$. Therefore the left hand side of inequality (5) is strictly greater than the right hand side for any positive probability, which contradicts our assumption.

iv) What we have just shown means that the players will not choose any contribution from the interval (a,z_S). Let us check that this is the case. Assume that all other players play according to the candidate equilibrium while player i contributes $g\in(a,z_S)$. Then i wins the prize with probability $(1-p)^{n[z_S]-1}=\frac{(\beta-1)a}{\beta\Pi}$ and his expected payoff is

$$\begin{aligned} & \beta z_S+\beta\Pi((\frac{(\beta-1)a}{\beta\Pi})^{\frac{1}{n[z_S]-1}})^{n[z_S]-1}-(\beta-1)g+G_{-i} \\ = \ & \beta z_S+(\beta-1)a-(\beta-1)g+G_{-i} \end{aligned}$$

which is strictly less than βz_S+G_{-i}. Therefore contributing 0 dominates any choice $g\in(a,z_S)$.

v) Let us now show that, when players with endowment z_S play according to the equilibrium candidate, it is a dominant strategy for all the other players to contribute nothing. Suppose that $z_{S-1}>a$. Point iv) proves that contributing 0 dominates any $g\in(a,z_{S-1}]$. On the other hand, if a player i with endowment $z_i<z_S$ contributes $g_i\in(0,a]$ then his expected payoff is

$$\beta z_i+\beta\Pi(\frac{(\beta-1)g_i}{\beta\Pi})^{\frac{n[z_S]}{n[z_S]-1}}-(\beta-1)g_i+G_{-i}$$

Given that

$$(\frac{(\beta-1)g_i}{\beta\Pi})^{\frac{n[z_S]}{n[z_S]-1}} < \frac{(\beta-1)g_i}{\beta\Pi}$$

it must be the case that contributing 0 is a dominant strategy for all players with endowment lower than z_S.

The same is true when $z_{S-1} \leq a$. ■

Finally, if the prize level is "low" only the players with endowments higher than $\frac{\beta\Pi}{\beta-1}$ will contribute positive amounts.

Proposition 4 *When $z_S \geq \frac{\beta\Pi}{\beta-1}$ and $n[z_S] > 1$, there exists a mixed strategy equilibrium in which:*

- *players with endowment $z_i \geq \frac{\beta\Pi}{\beta-1}$ choose their contributions from the distribution function $F(g) = (\frac{(\beta-1)g}{\beta\Pi})^{\frac{1}{m-1}}$ on the interval $[0, \frac{\beta\Pi}{\beta-1}]$, where m is the number of players with endowment greater or equal than $\frac{\beta\Pi}{\beta-1}$;*
- *all other players contribute* 0.

Proof. Suppose that $z_{l-1} < \frac{\beta\Pi}{\beta-1}$ while $z_l \geq \frac{\beta\Pi}{\beta-1}$, with $1 \leq l \leq S$, and call $m = \sum_{i=l}^{S} n[z_i]$ the number of players with endowment greater or equal than $\frac{\beta\Pi}{\beta-1}$. If $l = 1$ then consider z_{l-1} to be zero. The proof is in four parts.

i) Notice first that any strategy above $\frac{\beta\Pi}{\beta-1}$is dominated by contributing 0.

ii) Let us focus on the interval $(z_{l-1}, \frac{\beta\Pi}{\beta-1}]$ where only m players are active. Assume that all but one of the m players choose their contribution from the distribution function $F(g)$ on the interval $(z_{l-1}, \frac{\beta\Pi}{\beta-1}]$. In order for this to be an equilibrium the remaining player i must be indifferent to play any $g \in (z_{l-1}, \frac{\beta\Pi}{\beta-1}]$. Hence his expected payoff from playing g must be

$$\beta z_i + \beta\Pi(F(g))^{m-1} - (\beta-1)g + G_{-i} = \beta z_i + G_{-i} + c$$

where $c \geq 0$.

This means that on the interval $(z_{l-1}, \frac{\beta\Pi}{\beta-1}]$ any player with endowment greater than z_{l-1} randomises according to the following distribution function

$$F(g) = (\frac{(\beta-1)g + c}{\beta\Pi})^{\frac{1}{m-1}}$$

Note that $F(\frac{\beta\Pi}{\beta-1}) \leq 1$ implies that c must be equal to 0 and therefore we have a unique solution

$$F(g) = (\frac{(\beta-1)g}{\beta\Pi})^{\frac{1}{m-1}} \qquad (6)$$

iii) Suppose that $l = 1$. When the other $N-1$ players choose their contribution from $F(g)$ on the interval $[0, \frac{\beta\Pi}{\beta-1}]$, then player i's expected payoff is equal to

$$\beta z_i + G_{-i}$$

independently of his contribution on the same interval.

iv) If $l > 1$ then point v) of the proof of Proposition (3) shows that contributing 0 is a dominant strategy for all players with endowment less than $\frac{\beta\Pi}{\beta-1}$, while players with higher endowments will randomise according to $F(g)$ from the interval $[0, \frac{\beta\Pi}{\beta-1}]$. ■

Only One Player with the Highest Endowment

We look now at the case where $n[z_S] = 1$. First we will prove that only mixed strategy equilibria exist.

Proposition 5 *When $n[z_S] = 1$ there exist no pure strategy equilibria.*

Proof. The proof is in two parts.

i) Consider the case $z_{S-1} < \frac{\beta\Pi}{\beta-1}$. Suppose that there exists a pure strategy equilibrium characterised by the strategy profile $[g_1, \ldots, g_i, \ldots, g_N]$, where g_i is the contribution chosen by the generic player i. Call g_h the highest contribution. If $g_h > z_{S-1}$ then the player with endowment z_S could marginally lower his bid and increase his payoff. If $g_h < z_{S-1}$ then there is at least one player who could deviate and contribute $g_h + \varepsilon$, winning the prize and making a positive profit. If $g_h = z_{S-1}$ and a player with endowment z_{S-1} is contributing g_h, then the player with the highest endowment has an incentive to deviate and contribute $z_{S-1} + \varepsilon$. If $g_h = z_{S-1}$ and the players with endowment z_{S-1} are contributing strictly less than g_h, then the player with endowment z_S could lower his bid increasing his payoff.

ii) Consider the case $z_{S-1} \geq \frac{\beta\Pi}{\beta-1}$. Notice that any strategy $g > \frac{\beta\Pi}{\beta-1}$ is dominated by $g = 0$. As we have done above, suppose that there exists a pure strategy equilibrium characterised by the strategy profile $[g_1, \ldots, g_i, \ldots, g_N]$ and call g_h the highest contribution. If $g_h < \frac{\beta\Pi}{\beta-1}$ then there is at least one player who has an incentive to deviate and contribute $g_h + \varepsilon$. If $g_h = \frac{\beta\Pi}{\beta-1}$ and only one player is contributing g_h, then he could lower his bid. If $g_h = \frac{\beta\Pi}{\beta-1}$ and two or more players are bidding g_h, then each one of them would be better off by contributing zero. ■

There exist two possible cases: when the prize level is "low" and when it is "high". Let us start focusing on the first scenario.

Proposition 6 *When $z_{S-1} \geq \frac{\beta\Pi}{\beta-1}$ and $n[z_S] = 1$, there exists a mixed strategy equilibrium in which:*

- *players with endowment* $z_i \geq \frac{\beta\Pi}{\beta-1}$ *choose their contributions from the distribution function* $F(g) = (\frac{(\beta-1)g}{\beta\Pi})^{\frac{1}{m-1}}$ *on the interval* $[0, \frac{\beta\Pi}{\beta-1}]$*, where* m *is the number of players with endowment greater or equal than* $\frac{\beta\Pi}{\beta-1}$*;*

- *all other players contribute* 0*.*

Proof. Proof as in Proposition (4). ■

When the prize level is "high", specifically $z_{S-1} < \frac{\beta\Pi}{\beta-1}$, if the strategy space is continuous, and ties are broken by randomly assigning the prize to one player, then no equilibrium exists. In order to avoid this problem, given that we are interested in the theoretical predictions of an experiment, where the strategy space is discrete, we will assume that there exists a smallest currency unit strictly above z_{S-1} (see Che and Gale, 1997).[12]

Proposition 7 *When* $z_{S-1} < \frac{\beta\Pi}{\beta-1}$ *and* $n[z_S] = 1$*, there exists a mixed strategy equilibrium in which:*

- *the player with endowment* z_S *chooses his contribution from the distribution function* $H(g) = \frac{(\beta-1)g}{(\beta\Pi)^{\frac{1}{n[z_{S-1}]}}(\beta\Pi-(\beta-1)(z_{S-1}-g))^{\frac{n[z_{S-1}]-1}{n[z_{S-1}]}}}$ *on the interval* $[0, z_{S-1}]$ *and puts a mass equal to* $\frac{\beta\Pi-(\beta-1)z_{S-1}}{\beta\Pi}$ *on the smallest currency unit strictly above* z_{S-1}*;*

- *players with endowment* z_{S-1} *contribute zero with probability* $(\frac{\beta\Pi-(\beta-1)z_{S-1}}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}}$ *and choose their contribution from the distribution function*

$$L(g) = (\frac{\beta\Pi-(\beta-1)(z_{S-1}-g)}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}} \text{ on the interval } (0, z_{S-1}];$$

- *all other players contribute zero.*

Proof. Assuming that the players with budgets z_{S-1} and z_S are the only ones who submit positive bids, we show that by playing according to the equilibrium candidate they make each others indifferent between any possible choice. We then go on to prove that if they play in such a way it is a dominant strategy for all other players to contribute zero. The proof is in three parts.

i) Let us start supposing that the players with endowments strictly lower than z_{S-1} contribute zero. Note first that the player of type z_S can guarantee himself a positive surplus by submitting a bid above z_{S-1}. We want to show

[12]The non-existence of the equilibrium is due to a discontinuity in the payoffs. Another way to avoid this problem would be to always break ties in favour of the player with the higher budget.

that if players with endowment z_{S-1} choose their contribution from $L(g)$, and play zero with probability $(\frac{\beta\Pi-(\beta-1)z_{S-1}}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}}$, then the agent with the highest endowment is indifferent between any choice on the interval $(0, z_{S-1}]$. His payoff from playing $g \in (0, z_{S-1}]$ will be

$$\begin{aligned} & \beta z_S + \beta\Pi(L(g))^{n[z_{S-1}]} - (\beta-1)g + G_{-i} \\ = \; & \beta z_S + \beta\Pi(\frac{\beta\Pi - (\beta-1)(z_{S-1}-g)}{\beta\Pi}) - (\beta-1)g + G_{-i} \\ = \; & \beta z_S + \beta\Pi - (\beta-1)z_{S-1} + G_{-i} \end{aligned}$$

which indeed does not depend on g.

ii) Suppose now that the player with endowment z_S randomises according to $H(g)$ on the interval $[0, z_{S-1}]$ and puts a mass equal to $\frac{\beta\Pi-(\beta-1)z_{S-1}}{\beta\Pi}$ on the smallest currency unit strictly above z_{S-1}.[13] If all other agents of type z_{S-1} play according to $L(g)$, and contribute zero with probability $(\frac{\beta\Pi-(\beta-1)z_{S-1}}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}}$, then the payoff of a player with z_{S-1} from a choice $g \in [0, z_{S-1}]$ is given by

$$\begin{aligned} & \beta z_S + \beta\Pi(L(g))^{n[z_{S-1}]-1}H(g) - (\beta-1)g + G_{-i} \\ = \; & \beta z_S + \beta\Pi(\frac{\beta\Pi - (\beta-1)(z_{S-1}-g)}{\beta\Pi})^{\frac{n[z_{S-1}]-1}{n[z_{S-1}]}} \\ & \frac{(\beta-1)g}{(\beta\Pi)^{\frac{1}{n[z_{S-1}]}}(\beta\Pi - (\beta-1)(z_{S-1}-g))^{\frac{n[z_{S-1}]-1}{n[z_{S-1}]}}} \\ & -(\beta-1)g + G_{-i} \\ = \; & \beta z_S + G_{-i} \end{aligned}$$

which again is independent of g. It should be clear now why it is necessary to assume that there exists a smallest unit strictly above z_{S-1}. If this was not the case the player with the highest endowment would have a mass point at z_{S-1}. But then, if ties are broken by randomly assigning the prize to one player, an agent of type z_{S-1} would have an incentive to deviate and bid all his endowment.

iii) Finally, we want to show that if the agents of type z_{S-1} and z_S play as we described then it is a dominant strategy for all other players to contribute zero. If a player i with endowment $z_i < z_{S-1}$ contributes $g \in (0, z_i]$ his payoff is represented by

$$\beta z_S + \beta\Pi(L(g))^{n[z_{S-1}]}H(g) - (\beta-1)g + G_{-i} \tag{7}$$

[13] Note that, acording to $H(g)$, player z_S's bid is strictly positive and therefore no ties are possibile at zero.

$$
\begin{aligned}
&= \beta z_S + \beta\Pi(\frac{\beta\Pi - (\beta-1)(z_{S-1}-g)}{\beta\Pi}) \\
&\quad \frac{(\beta-1)g}{(\beta\Pi)^{\frac{1}{n[z_{S-1}]}}(\beta\Pi - (\beta-1)(z_{S-1}-g))^{\frac{n[z_{S-1}]-1}{n[z_{S-1}]}}} - (\beta-1)g + G_{-i} \\
&= \beta z_S + (\frac{\beta\Pi - (\beta-1)(z_{S-1}-g)}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}}(\beta-1)g - (\beta-1)g + G_{-i} \\
&= \beta z_S + (\beta-1)g((\frac{\beta\Pi - (\beta-1)(z_{S-1}-g)}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}} - 1) + G_{-i}
\end{aligned}
$$

On the other hand, if he plays $g = 0$ he gets a payoff equal to $\beta z_S + G_{-i}$. Note that $(\frac{\beta\Pi-(\beta-1)(z_{S-1}-g)}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}} < 1$ and we conclude that expression (7) is strictly lower than $\beta z_S + G_{-i}$. ■

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